

Equations of Motion

Defining Symbols

The following letters are assigned to represent each variable:

| | | | |
|------------------|-------|------------------|------------------------------|
| Displacement | = s | m | metres |
| Initial Velocity | = u | m/s | metres per second |
| Final Velocity | = v | m/s | metres per second |
| Acceleration | = a | m/s ² | metres per second per second |
| Time | = t | s | seconds |

Equations of Motion

Equation 1

If we start with the equation for acceleration $a = \frac{(v-u)}{t}$ we can rearrange this to give us an equation 1

$$at = (v-u) \rightarrow at + u = v$$

$$v = u + at$$

Equation 2

We start with the definition of velocity and rearrange for displacement
velocity = displacement / time \rightarrow displacement = velocity x time

In situations like the graph to the right the velocity is constantly changing, we need to use the average velocity.

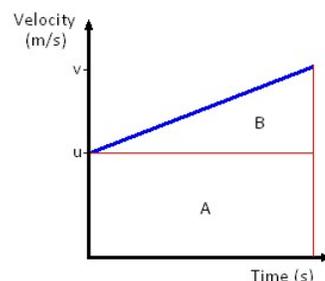
$$\text{displacement} = \text{average velocity} \times \text{time}$$

The average velocity is give by: average velocity = $\frac{(u+v)}{2}$

We now substitute this into the equation above for displacement

$$\text{displacement} = \frac{(u+v)}{2} \times \text{time} \rightarrow s = \frac{(u+v)}{2}t$$

$$s = \frac{1}{2}(u+v)t$$



Equation 3

With Equations 1 and 2 we can derive an equation which eliminated v . To do this we simply substitute

$$v = u + at \text{ into } s = \frac{1}{2}(u+v)t$$

$$s = \frac{1}{2}(u + (u + at))t \rightarrow s = \frac{1}{2}(2u + at)t \rightarrow s = \frac{1}{2}(2ut + at^2)$$

$$s = ut + \frac{1}{2}at^2$$

This can also be found if we remember that the area under a velocity-time graph represents the distance travelled/displacement. The area under the line equals the area of rectangle A + the area of triangle B.

Area = Displacement = $s = ut + \frac{1}{2}(v-u)t$ since $a = \frac{(v-u)}{t}$ then $at = (v-u)$ so the equation becomes

$$s = ut + \frac{1}{2}(at)t \text{ which then becomes equation 3}$$

Equation 4

If we rearrange equation 1 into $t = \frac{(v-u)}{a}$ which we will then substitute into equation 2:

$$s = \frac{1}{2}(u+v)t \rightarrow s = \frac{1}{2}(u+v) \frac{(v-u)}{a} \rightarrow as = \frac{1}{2}(u+v)(v-u) \rightarrow$$

$$2as = (v^2 + uv - uv - u^2) \rightarrow 2as = v^2 - u^2$$

$$v^2 = u^2 + 2as$$

Any question can be solved as long as three of the variables are given in the question.

Write down all the variables you have and the one you are asked to find, then see which equation you can use.

*These equations can only be used for motion with **UNIFORM ACCELERATION**.*