

Ideal Gases

Combining the gas laws

The three gas laws can be combined to give us the relationship: $pV \propto T$

We can rearrange this to give the expression: $\frac{pV}{T} = \text{constant}$

We can use this to derive a very useful equation to compare the pressure, volume and temperature of a gas that is changed from one state (p_1, V_1, T_1) to another (p_2, V_2, T_2).

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

Temperatures must be in Kelvin, K

Avogadro and the Mole

One mole of a material has a mass of M grams, where M is the molecular mass in atomic mass units, u. Oxygen has a molecular mass of 16, so 1 mole of Oxygen atoms has a mass of 16g, 2 moles has a mass of 32g and so on. An Oxygen molecule is made of two atoms so it has a molecular mass of 32g. This means 16g would be half a mole of Oxygen molecules.

$$n = \frac{m}{M}$$

where n is the number of moles, m is the mass and M is the molecular mass.

Avogadro suggested that one mole of any substance contains the same number of particles, he found this to be 6.02×10^{23} . This gives us a second way of calculating the number of moles

$$n = \frac{N}{N_A}$$

where N is the number of particles and N_A is the Avogadro constant.

N_A is the Avogadro Constant, $N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$

Ideal Gases

We know from the three gas laws that $\frac{pV}{T} = \text{constant}$

Ideal gases all behave in the same way so we can assign a letter to the constant. The equation becomes:

$$\frac{pV}{T} = R$$

If the volume and temperature of a gas are kept constant then the pressure depends on R and the number of particles in the container. We must take account of this by bringing the number of moles, n , into the equation:

$$\frac{pV}{T} = nR \quad \rightarrow \quad pV = nRT$$

R is the Molar Gas Constant, $R = 8.31 \text{ J K}^{-1} \text{ mol}^{-1}$

This is called the *equation of state* for an ideal gas. The concept of ideal gases is used to approximate the behaviour of real gases. Real gases can become liquids at low temperatures and high pressures.

Using the Avogadro's equation for n we can derive a new equation for an ideal gas:

$$pV = nRT \quad \rightarrow \quad pV = \frac{N}{N_A} RT \quad \rightarrow \quad pV = N \frac{R}{N_A} T$$

Boltzmann Constant

Boltzmann noticed that R and N_A in the above equation are constants, so dividing one by the other will always give the same answer. The Boltzmann constant is represented by k and is given as

$$\frac{R}{N_A} = k$$

k is the Boltzmann Constant, $k = 1.38 \times 10^{-23} \text{ J K}^{-1}$

$$pV = nRT \text{ can become } pV = N \frac{R}{N_A} T \text{ which can also be written as: } pV = NkT$$