Topic 11 : Electromagnetic Induction

11.1 - Electromagnetic induction

Date

Essential Idea: Majority of electricity is produced by machines using the principles of electromagnetic induction.

Understandings

- Electromotive force (emf)
- Magnetic flux & magnetic flux linkage
- Faraday's Law of Induction
- Lenz's law

Guidance

- Quantitative treatments for straight conductors, rectangular coils rotating in fields, etc.
- Qualitative treatments for fixed coils in a changing magnetic field and ac generators

Applications and Skills

- Describing production of induced emf by a changing magnetic flux & within a uniform magnetic field
- Solving problems with magnetic flux, magnetic flux linkage and Faraday's Law
- Explaining Lenz law through conservation of energy

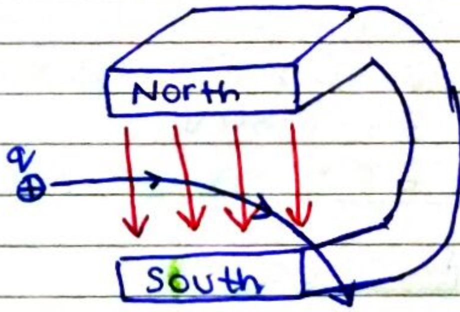
Data Booklet Reference

$\Phi = BA\cos\theta$ $\qquad \varepsilon = BVl$

$\varepsilon = \dfrac{-N\Delta\Phi}{\Delta t}$ $\qquad \varepsilon = BVlN$

1) Electro~~magnetic~~motive Force (emf)

1. Consider the B-field provided by a horseshoe magnet.
2. If we place a stationary charge q within B-field, it will ~~pell~~ feel NO MAGNETIC FORCE.
3. But, if we project the charge q through B-field with velocity v, it feels a force:



$F = qvB\sin\phi$ where ϕ is angle between v and B	Magnetic force on a moving charge

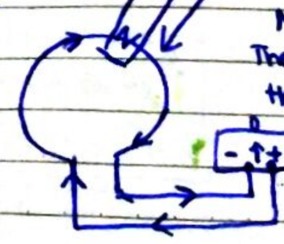
2) Induced Electromotive Force (emf)

When a wire is moved through the magnetic field instead, there is a magnetic force on the charges in the moving wire~~s~~. Flemings LHR, FBI

- If a north pole of a magnet is thrusted through a looped conductor, a current is created.

T or F: Current travels through circuit only when magnet moves through the loop.

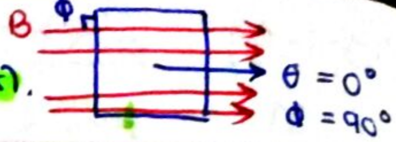
T or F: Current direction depends on which direction the magnet is moved through the loop.



Makes sense! The coil ~~opposes~~ repels the magnet by creating its own pole. Think!

KIKY

ϕ considers Area (plane).
θ considers direction of plane (perpendicular).



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Since Moving a conductor through a magnetic field produces a current, this very action induces an emf in the conductor.

All we need is relative motion in order to ~~pro~~ induce an emf.

FYI
This is the principle behind turbines & generators. Motion of a conductor through a B-field produces an EMF which can drive a current.

3) Induced electromotive force - straight wire through B

$\mathcal{E} = Bvl$	l = length (v and B are perpendicular)	Induced emf in a straight wire	ϕ

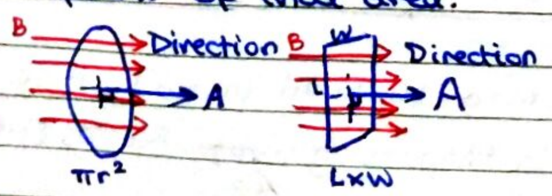
PRACTICE
Boeing Dreamliner, has wingspan of 60m, flies through Earth's magnetic field near Tuscon (place) ($B = 56\mu T$) at $265\,ms^{-1}$. Treating wingspan as straight line, Find the induced emf from wingtip to wingtip. Wingspan is measured from tip to tip.

Solution
$\mathcal{E} = (56\times10^{-6}) \times 265 \times (60)$
$\mathcal{E} = 0.89\,V$
$\mathcal{E} = 0.90\,V$

4) Magnetic Flux

When B-field is parallel to the plane of the loop, no B-field lines pass through it and so no emf or current induced. ϕ

The direction of a cross-sectional area is perpendicular to the plane of that area.



As $\theta = 90°$, $\Phi \to 0$. Φ = Magnetic flux
As $\theta = 0°$, $\Phi \to$ max.

$\Phi = BA\cos\theta$	Magnetic flux	Weber Wb
θ is angle between A & B		

$1\,Wb = 1\,Tm^2$

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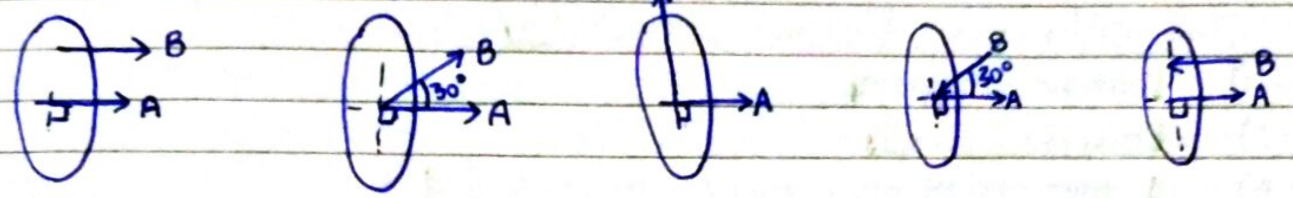
PRACTICE

Find the magnetic flux in each case. B-field strength is 1·5 T and area is $0{\cdot}20\,m^2$.

BA | $BA \times \frac{\sqrt{3}}{2}$ | $BA\cos 90 = 0$ | $BA\cos 150°$ | $BA\cos 180°$



$\Phi = BA\cos\theta$
$\Phi = 1{\cdot}5 \times 0{\cdot}20 \times \cos 0$
$= 0{\cdot}30$ Weber

$\Phi = BA\cos\theta$
$\Phi = 1{\cdot}5 \times 0{\cdot}2 \times \cos 30$
$= 0{\cdot}26$ Wb

$\Phi = BA\cos\theta$
$\Phi = 1{\cdot}5 \times 0{\cdot}2 \times \cos 90$
$\Phi = 0$ Wb or Tm^2

$\Phi = BA\cos\theta$
$\Phi = 1{\cdot}5 \times 0{\cdot}2 \times \cos 210$
$\Phi = 1{\cdot}5 \times 0{\cdot}2 \times \cos 150$
$\Phi = -0{\cdot}26$ Wb

$\Phi = BA\cos\theta$
$\Phi = BA\cos 180$
$\Phi = -0{\cdot}30$ Wb

Φ can be negative.

Exam Tip:
For solar radiation flux:
$$\Phi = IA\cos\theta$$
where I = intensity = $1380\,Wm^{-2}$

5) Magnetic Flux Density

Magnetic flux Density = Magnetic flux per unit area.

$$\text{Magnetic Flux Density} = \frac{\Phi}{A\cos\theta} = \frac{BA\cos\theta}{A\cos\theta}$$

Magnetic flux density = B

Tip!
The Magnetic flux density and Magnetic field strength are the same thing - namely, the B-field.

6) Magnetic Flux Linkage

- Each loop produces an emf that is added to total induced emf. EMF is only produced when flux is changing.

Flux Linkage = $N\Phi$ = $NBA\cos\theta$	N is no. of loops	Flux linkage

Each loop is "linked".

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7) Induced Electromotive Force

So we know that, only when flux is changing, emf is produced.

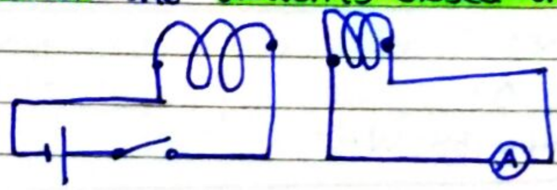
The only ways to change the flux is:

1) Change B-field.
2) Change Area A.
3) Change relative orientation θ of A & B.

Usually, 3) is used at power plant, where a turbine rotates the coils, thus changing θ.

EXAMPLE

Why does the ammeter in second circuit read a current for an instant when the switch is closed in the first circuit.



- When circuit 1 is closed, current flows and a B-Field is produced.
- When it was open, there was no B-field. It grows from 0 to CONST.
- While it is growing, it is changing, and the changing B-field causes change in magnetic flux is detected by circuit 2, thus inducing an EMF. A current is detected.
- Once the B-field is CONST, there is no more change in magnetic flux, and the current and emf drop to 0.

8) Faraday's Law of induction and Lenz's Law

- Faraday's law - the emf induced in a coil is equal OR proportional to the rate of change ~~in the~~ of magnetic flux linkage in the coil.

$\varepsilon = -\dfrac{N\Delta\Phi}{\Delta t}$	Faraday's Law	N = number of loops (turns).

- Lenz's law - an induced current will have a direction such that it will oppose the change in magnetic flux that produced it. (This explains the - sign in Faraday's law.)

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Practice: Suppose magnetic flux in 240 loops is changing at a rate of 0.25 $Wb s^{-1}$. What is the induced emf?

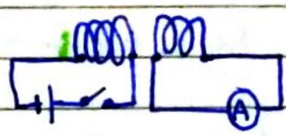
$$\frac{\Delta \Phi}{\Delta t} = 0.25 Wb s^{-1}$$

$$\mathcal{E} = -N \times 0.25 = -240 \times 0.25$$

$$\mathcal{E} = -60V$$

Ignore the (-) if direction is not needed.

Example

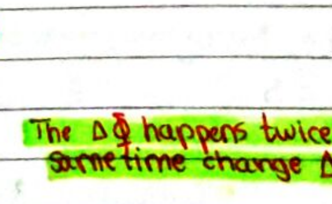


Explain why when switch is closed in first circuit, induced current produces a flux change that is opposite to the original flux linkage.

Solution

Suppose, induced flux change were NOT opposite.
Then, the induced flux change ADDs to original, eventually leading to infinite induced current.
It violates conservation of energy.

Lenz's Law is a statement of conservation of energy.



Q) The maximum voltage in this experiment is 18V.
a) What happens if south pole enters first?
Ans) Magnet reversal → $\cos(180+\theta) = -\cos\theta$.
Sign of flux reverses and meter will deflect in other direction.

b) What is the effect of doubling oscillation speed of magnet?
Ans) $\mathcal{E} = -\frac{N\Delta\Phi}{\Delta t} = 18V$. $\Delta t \rightarrow \frac{1}{2}\Delta t$, $18 \times 2 = 36V$
Since Δt is halved, emf doubles to 36V. The $\Delta\Phi$ happens twice in the same time change Δt.

c) At original oscillation rate, what would you predict voltage induced in a single loop to be?
Ans) We see 7 loops. $\mathcal{E} = \frac{18}{7} = 2.6V$ per loop.

d) If 150 loops, what is the voltage?

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t} = 150 \times 2.6 = 390V$$

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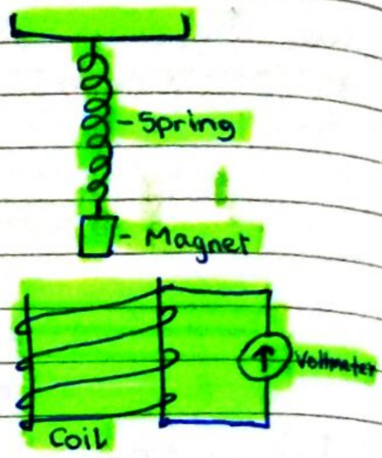
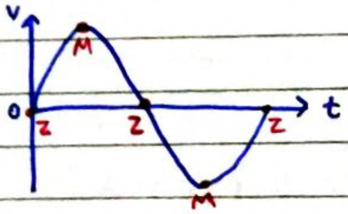
8) Electromagnetic Induction Problems

Date

Q) A bar magnet is suspended as shown. The bar magnet is pulled down so that its north pole is level with the top of the coil.



The magnet is released and the variation with time t of the velocity v of the magnet is shown below.



a i) Mark with M, point where reading on voltmeter is maximum.

Ans. ~~Emf is max~~ ε is max when $\frac{\Delta\Phi}{\Delta t}$ is max. $\left(\frac{\Delta\Phi}{\Delta t}\right)$ Rate of Change in flux is max when velocity of oscillation is ↑.

Hence, the velocity must be maximum.

ii) Mark with Z, point where voltage reading is 0.

$\varepsilon = -N\frac{\Delta\Phi}{\Delta t}$. When magnet is not moving, $\Delta\Phi$ is 0. $v = 0$.

b) Explain, in terms of changes in flux linkage, why the voltmeter reading is alternating.

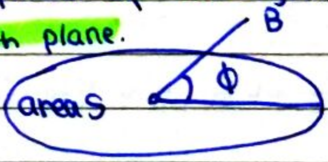
✓ • The magnet oscillates due to the spring. Good to read!

✓ • Hence, B-field is oscillating (changing), and so is the magnetic flux linkage.

✓ • Hence, the induced emf also oscillates since the rate of change of magnetic flux linkage is changing ~~Faraday's Law~~ and it is equal to the induced ~~current~~ emf. Faraday's Law.

$\frac{\Delta\Phi}{\Delta t}$ is changing.

Q) A uniform B-field of strength B links a coil of area S. Field makes Φ angle with plane.

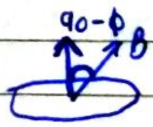


IBO is a sneaky c*nt. Beware of these things, seriously!

The magnetic flux linking the coil is:

A. BS B. $BS\cos\phi$ C. $BS\sin\phi$ D. $BS\tan\phi$

Remember? We need direction of S.



$$\Phi = BS\cos(90-\phi) = BA\sin\phi$$

$\boxed{\text{Answer. C}}$

Date

Q) Faraday's Law states that the induced emf is:

A. Proportional to change in magnetic flux linkage.
B. Proportional to rate of change of magnetic flux linkage.
C. Equal to the change in magnetic flux linkage.
D. Equal to change of magnetic flux.

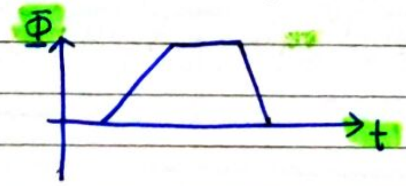
$\mathcal{E} = -\frac{N \Delta \Phi}{\Delta t}$ $N\Delta\Phi$ = Magnetic flux linkage change

Δt = rate

A, D, C are out.

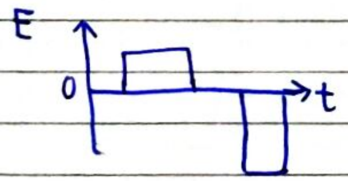
Answer: B

Q)

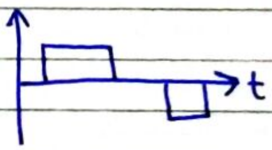


Which graph shows variation with time t of emf E induced in coil?

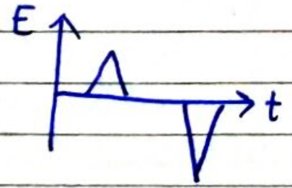
A.



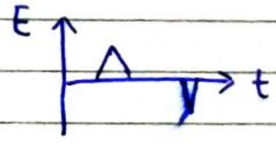
B.



C.



D.



$\mathcal{E} = E = \frac{-\Delta\Phi}{\Delta t}$, gradient of the Φ by t graph.

Gradient is constant at every slope. So, C and D are wrong.
More steep when Φ decreases. So $\frac{\Delta\Phi}{\Delta t}$ is higher. Hence, higher E induced.

Answer: A

Date

Q) A thin copper ring encloses area S. Area is linked by magnetic flux that is increasing. Rate of change of magnetic flux from $t=0$ to $t=T$ is R. What is the emf induced from $t=0$ to $t=T$?

A. R B. RS C. RST D. $\frac{RS}{T}$

~~$\Delta t = T$~~ ~~$\frac{\Delta\Phi}{\Delta t} = R$~~ ~~$\Delta\Phi = R$~~ $\Delta\Phi$ $\Delta t = T - 0 = T$

$N = 1$ $\frac{\Delta\Phi}{T} = R = EMF$

~~Ans A R~~ $\boxed{\text{Ans. A}}$

Read carefully. This question tried to trick you.
emf induced = rate of change of magnetic flux, Faraday

(KIKY)

11.2 - Power Generation and Transmission

Date

Essential Idea: Generation and transmission of alternating current (ac) has transformed the world.

Understandings

- Alternating current (ac) generators
- Average power & root mean square (rms) values of current & voltage
- Transformers
- Diode bridges
- Half-wave & full-wave rectification

Guidance

- Some reasons why transformers are not ideal are: flux leakage, eddy current heating, joule heating, magnetic hysteresis

Applications & skills

- Explaining operation of an ac generator, including effect of changing generator frequency.
- Solve problems involving average power in ac circuit.
- Solve problems involving step-up &-down transformers.
- Describe use of transformers in an ac electrical power distribution.
- Investigating diode bridge rectification circuit experimentally
- Qualitatively describing effect of adding a capacitor to a diode bridge rectification circuit.

Data Booklet Reference

- $I_{rms} = \dfrac{I_0}{\sqrt{2}}$
- $V_{rms} = \dfrac{V_0}{\sqrt{2}}$
- $R = \dfrac{V_0}{I_0} = \dfrac{V_{rms}}{I_{rms}}$
- $P_{max} = I_0 V_0$
- $\bar{P} = \frac{1}{2} I_0 V_0$
- $\dfrac{\varepsilon_p}{\varepsilon_s} = \dfrac{N_p}{N_s} = \dfrac{I_s}{I_p}$

1) Alternating current (ac) generators

Remember Faraday's Law? $\varepsilon = -N \dfrac{\Delta\Phi}{\Delta t}$

To increase induced emf: ① Increase number of coils
② Increase change in magnetic flux
③ Decrease change in time over which flux changes.

Since $\Phi = BA\cos\theta$, we can recall that as magnetic field is uniform, and so is rotating coil área, the only way of increasing $\Delta\Phi$ is by changing θ, angle.

So, to increase induced emf ε, we usually increase $\Delta\Phi$, which is increased by changing angle θ between the direction (perpendicular) of Area of coil and the magnetic field B.

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PRACTICE

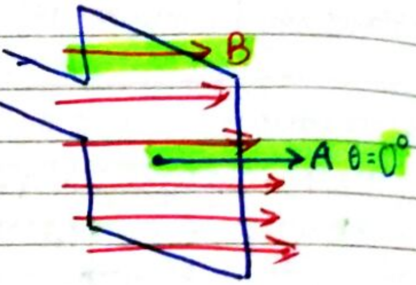
What is the effect of increasing the frequency of the generator on the induced emf?

or $\Delta\Phi$ increases over Δt

By increasing frequency, Δt decreases. Thus, increasing the frequency increases the induced emf.

Very important understanding:

Consider the rectangular loop that rotates in the fixed magnetic field:

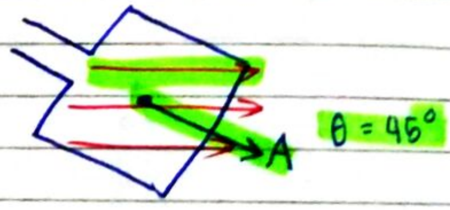


At this instant

$\Phi = BA\cos 0 = {}^{+}BA$

When θ changes to 45°,

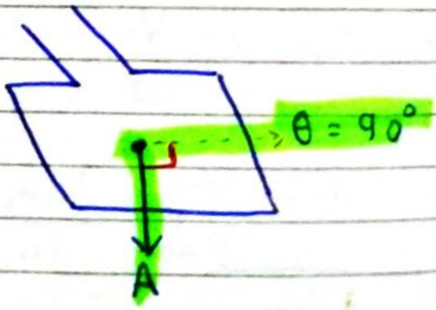
$\Phi = BA\cos 45 = {}^{+}0.7BA$

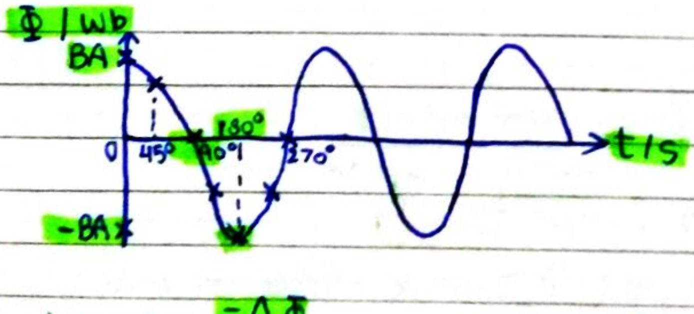


So, magnetic flux changes as coil rotates.

when θ changes to 90°,

$\Phi = BA\cos 90 = 0$

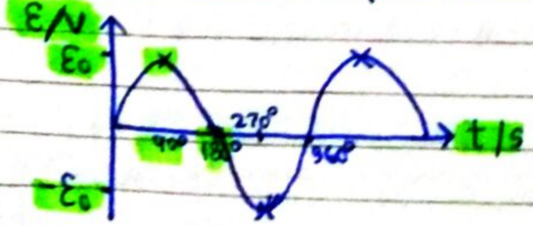




Sinusoidal pattern emerges.

Now ~~sine~~ since $\mathcal{E} = \frac{-\Delta\Phi}{\Delta t}$, we see that the $\mathcal{E}$ is the slope.

Slope of cosine plot is sine plot. We see that $\mathcal{E} = BA\sin\theta$.



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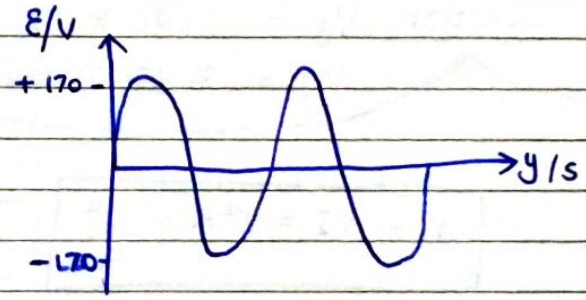
- The schematic for an AC supply is —o~o—.

So, basically, in a simple one-loop generator:

- Some mechanical means makes the coil rotate (a steam turbine or paddle wheels.)
- The alternating current is picked up by brushes. ~~Alternating current means~~
The ~~the~~ magnetic flux continuously changes, which induces emf.
since coil rotates

2) Root mean square (rms) values of current and voltage

American power plants produce ~~170V~~ 120V AC, but if you record the voltage over-time, it varies between −170V & 170V.



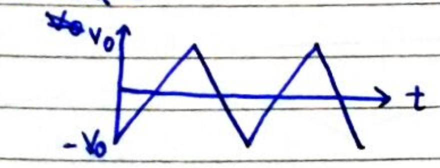
Average = 0V, sinusoidal function
So, we need different method.

Root Mean Square

Square → Mean(÷2) → Root (Square)

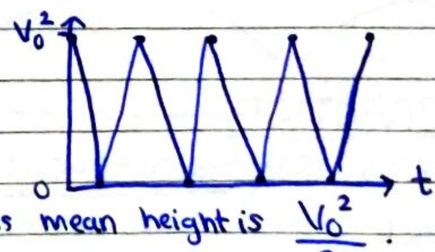
- The AC rms values are same to the constant DC values that would dissipate the same power.

Example



Find the rms voltage for the triangular wave.

① Square the graph.



② Divide by 2 as mean height is $\frac{V_0^2}{2}$.

③ Square root. $\sqrt{\frac{V_0^2}{2}}$

$$V_{rms} = \frac{V_0}{\sqrt{2}}.$$

$V_{rms} = \frac{V_0}{\sqrt{2}}$ (V_0 = peak voltage)	root mean square (rms) of an alternating voltage

Try for the example of USA above!

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A current set by alternating voltage similarly alternates.

$I_{rms} = \frac{I_0}{\sqrt{2}}$ I_0 = peak current	Root mean square of an alternating current (ac)

rms values ~~are~~ of AC are equivalent to power dissipation in DC.

Example

Q) "Mains" (outlet) electricity in the UK is rated at 220V. This is an rms value. Find peak voltage.

Solution $V_0 = 220 \times \sqrt{2}$

$V_0 = 310V$

$$\boxed{P = VI = I^2R = \frac{V^2}{R}}$$ Electrical Power

The Power consumption depends on rms, not peak voltage & current. Hence, in ac alternating supply ~~currents~~,

$P = V_{rms} I_{rms} = I_{rms}^2 R = \frac{V_{rms}^2}{R}$	Electrical Power (AC)

PRACTICE

Show that for an AC circuit $P = \frac{1}{2} V_0 I_0$.

Sol.

$P = V_{rms} I_{rms}$

$P = \frac{V_0}{\sqrt{2}} \times \frac{I_0}{\sqrt{2}} = \frac{1}{2} V_0 I_0$

Date

3) Alternating current (ac) generators

Question: An AC generator produces a peak voltage V. Frequency of rotation of coil is doubled. The rms value of the voltage produced is:

A. $\frac{V}{2\sqrt{2}}$ B. $\frac{V}{\sqrt{2}}$ C. $V\sqrt{2}$ D. $2V\sqrt{2}$

As frequency doubles, period T halves.

$\Delta t \rightarrow \frac{1}{2}\Delta t$

$$\mathcal{E} = \frac{\Delta\Phi}{\frac{1}{2}\Delta t} = 2\frac{\Delta\Phi}{\Delta t}$$

So, the emf doubles.

$\mathcal{E}$ is the rms value $\rightarrow \frac{V}{\sqrt{2}}$

Doubling $\mathcal{E} \rightarrow \frac{2V}{\sqrt{2}} = \sqrt{2} \times V = V\sqrt{2}$

Answer: C. $V\sqrt{2}$

4) Solve AC circuit problems for ohmic resistors

$V_{rms} = I_{rms} R$ $V_0 = I_0 R$	ac circuits & resistors

- This means that V and I are proportional & in phase for ohmic resistors.

Good Question!

Max value of a sinusoidal alternating current in a resistor of resistance R is I_0. The Max current is increased to $2I_0$. Assuming that the resistance of resistor remains constant, the average power dissipated in the resistor is now:

Sol. $P = I_{rms}^2 R$

$I_{rms} = \sqrt{\frac{(2I_0)^2}{2}} = \sqrt{2}\, I_0$

$P = 2I_0^2 R$

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Example

A 60 Hz generator produces max value of $\mathcal{E}_0 = 140V$. It is connected to a resistor of $28\,\Omega$ as shown. Find the equation of the current I as a function of time.

$$V_0 = I_0 R$$
$$140 = I_0(28)$$
$$I_0 = 5\,A$$

From $I = I_0 \sin 2\pi f t$,

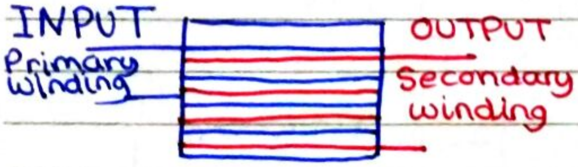
we have $I = 5.0 \sin 120\pi t$.

5) Transformers

A transformer is a device that changes a higher AC voltage to a lower one (step down) or a lower AC ~~voltage~~ voltage to a higher one (step up).

How a transformer works?

1) The input coil, primary winding, is wrapped around a soft iron core.
2) The output coil, secondary winding, is wrapped around the same soft iron core, but with different number of turns.
3) The loop ratios determine the transformer type.



Ideal Transformer

- No power loss. $\rightarrow \quad I_p V_p = I_s V_s \rightarrow \dfrac{I_p}{I_s} = \dfrac{V_s}{V_p}$

- Because of flux linkage we know that voltage is proportional to number N of loops.

Therefore, $V_p \propto N_p$, $V_s \propto N_s$

$$\frac{V_p}{V_s} = \frac{N_p}{N_s}$$

$\dfrac{I_s}{I_p} = \dfrac{V_p}{V_s} = \dfrac{N_p}{N_s}$	Ideal Transformer in DB

- To enhance Flux linkage between primary & secondary coils, transformers are made from iron ore.
- The iron core is laminated (insulated from other layers) to reduce eddy current and hysterisis current energy losses.  Iron core layers laminated.
- The primary and secondary coils are often concentric. (Secondary coil surrounds primary coil)
  - → Reduces Flux leakage

Real Transformers

- These suffer from eddy currents and hysteresis currents, both of which are created by Faraday's law due to magnetic flux change. They both produce $P = I^2 R$ heat loss, where I is the ~~type of~~ current.

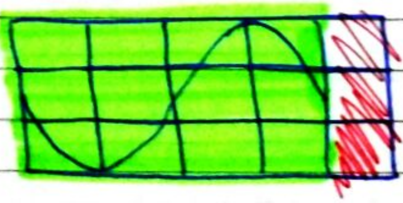
- Hysteresis losses ($I_{HYST} \propto f$) < Eddy losses ($I_{eddy} \propto f^2$)

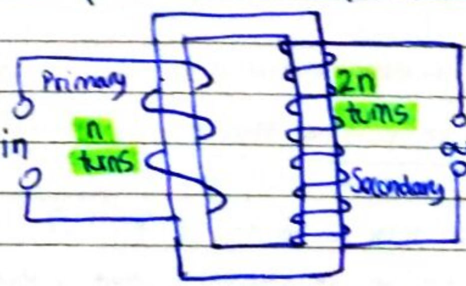
  Less significant — More significant

- Eddy currents are minimized by lamination of the transformer core.

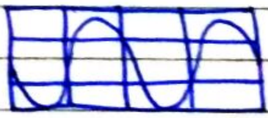
Good Question

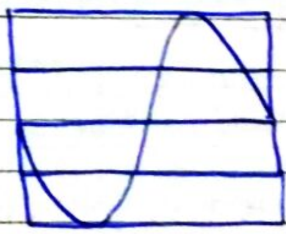
Q) The diagram shows an ideal transformer. The ~~wavefront~~ waveform connected to the primary coil, is shown.

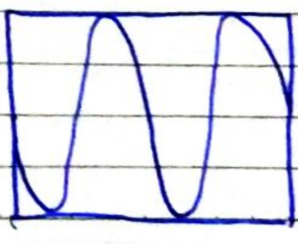




Which diagram shows waveform of secondary coil?

A. 

B. 

C. 

D. 

Sol. $\frac{N_p}{N_s} = \frac{V_p}{V_s}$

$\frac{1}{2} = \frac{V_p}{V_s}$

$V_s = 2V_p$

~~Magnitude or~~ amplitude is ~~big~~ doubled.

A and D X.

Transformers don't change Frequency.

C is wrong.

$\boxed{\text{Answer. B}}$

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6) Transformers in AC electrical power distribution

Generation → Transmission → Distribution

- Power is lost → ① During step-up & step-down of voltage, due to eddy currents. ($I_{eddy} \propto f^2$)
- Power is lost → ② in transmission lines due to internal resistance ($P = I^2R$).

There is a trade-off between large-diameter (low R) costly wires & low diameter (high R) wire.

7) Solve problems involving power transmission

Like reducing power loss etc

① Heat loss is determined by I^2R.

② Since R is fixed once diameter of cable is fixed, we can only reduce power loss by reducing current I.

③ As $P = VI$, if we increase V, we reduce I.

④ This is the idea behind the step-up transformer at the generation side (before transmission lines) of the power grid.

Generation Station → Step-up transformer → Transmission Lines → Step-down transformer → Customer

⑤ Since the high voltage is very dangerous, it is reduced by a step-down transformer at the end of the transmission line.

Q) The 150 km cable whose resistance is 1Ω is designed to deliver 270 MW to houses.

a) If transmission occurs at 138 kV, what is the current and the heat loss?

Sol. $P = VI$

$$\frac{270 \times 10^6}{138 \times 10^3} = I = 2000\,A \ (1957\,A)$$

$$P = I^2R = 1957^2 \times 1 = 3.8 \times 10^6\,W$$

Heat loss = 3.8 MW

b) If transmission occurs at 765 kV, what is the current & what is the heat loss?

$P = VI$

$$\frac{270 \times 10^6}{765 \times 10^3} = I = 350\,A \ (353\,A)$$

Step-up: The higher voltage reduces current and thus power loss.

$$P = I^2R = 353^2 \times 1 = 1.2 \times 10^5\,W$$

$$= 0.12\,MW$$

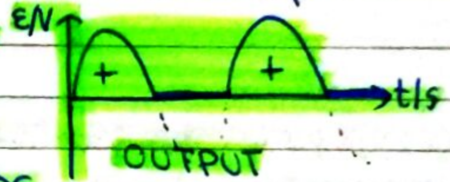
8) The AC vs DC argument

+ AC can be stepped up & down. Hence, more preferable for transmission.
+ Easier to produce AC in quantity.

9) The diode bridge rectifier

Diode : a semiconductor that allows current to flow only one-way. —▷|—
It symbol shows the direction of the conventional (+) current flow.

If we supply an AC input, only the positive lobes will pass through.

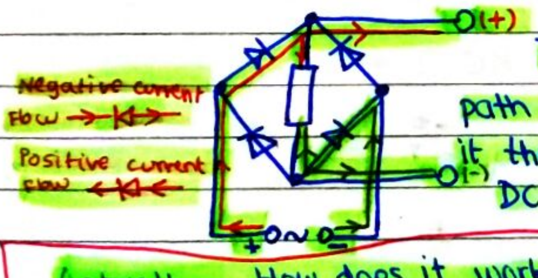


Rectification - the conversion from AC to DC.

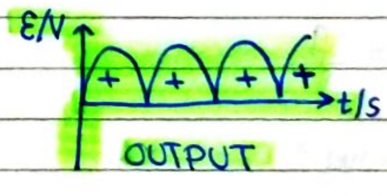
AC input → DC output

This graph shows a half-wave rectifier since we only get half of the electrical energy.

Full-wave bridge rectifier



There is always a path for a lobe to make it through & produce a DC voltage.



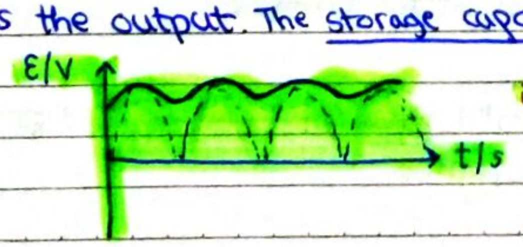
Actually, How does it work?
The + flows with the diode arrow,
and - flows against the diode arrow.

AC input → DC output

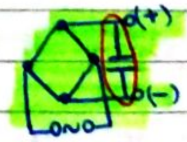
We will always have a DC output.

Now the problem is that the voltage fluctuation is too great. 0 to V_0 twice per period.

This is corrected through a large-valued capacitor across the output. The storage capacity "smooths" the waveform.



Not going to 0 ~~again~~ anymore.



11.3 - Capacitance

Date

Essential idea: Capacitors can store electricity energy for later use.

Understandings

- Capacitance
- Dielectric Materials
- Capacitors in series & parallel
- Resistance-capacitor (RC) series circuits
- Time constant

Guidance

- Only single parallel-plate capacitors with uniform magnetic field, with ~~a~~ load, are considered.
- Problems of discharge through fixed resistors are treated both graphically & algebrically
- Problems with charging are only graphic.

Applications & Skills

- Describing effect of different dielectric materials on capacitance.
- Solve problem with parallel-plate capacitors
- Investigation of series and parallel circuits
- Determine stored energy in a charged capacitor
- Describe nature of exponential discharge of a capacitor
- Solving problems involving discharge through a fixed resistor
- Solving problems involving time constant of an RC circuit for charge, voltage & current.

Data Booklet

- $C = \frac{q}{V}$
- $C_{parallel} = C_1 + C_2 + \dots$
- $\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$
- $C = \frac{\varepsilon A}{d}$
- $E = \frac{1}{2}CV^2$
- $\tau = RC$
- $q = q_0 e^{-\frac{t}{\tau}}$
- $I = I_0 e^{-\frac{t}{\tau}}$
- $V = V_0 e^{-\frac{t}{\tau}}$

1) Capacitance

- Capacitor - An electronic device that stores charge.
- Basically, it is made of 2 separated parallel plates. In between, we have a non-conductive material called ~~die~~ dielectric.

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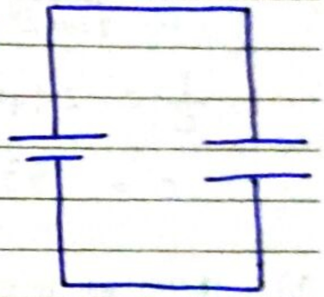
Date

- **Capacitance C** – the charge q per unit Voltage V which the capacitor is capable of maintaining.

$C = \frac{q}{V}$	Definition of Capacitance	CV^{-1} = Farad = F Units

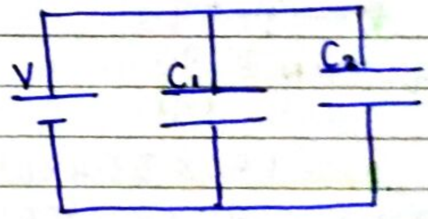
Example : A 1.50V cell is connected to a 275 μF capacitor in the circuit shown. How much charge is stored on the capacitor's plates?



Sol. $C = \frac{q}{V}$, $q = CV$

$$q = 275 \times 10^{-6} \times 1.50 = 4.13 \times 10^{-4}\ C$$

2) Capacitance – parallel



- Since they are connected in parallel, they both have the cell's voltage V.

$$V = V_1 = V_2$$

$$q = q_1 + q_2 ,$$

$$CV = C_1V_1 + C_2V_2$$

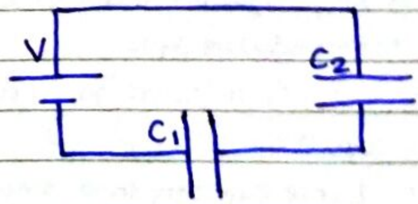
$$C\cancel{V} = C_1\cancel{V} + C_2\cancel{V}$$

$$C = C_1 + C_2$$

$C_{parallel} = C_1 + C_2 + \ldots$	Parallel Capacitance

Capacitance is higher in parallel than series

3) Capacitance - series



$q = q_1 = q_2$, Same charge q.

$V = \frac{q}{C}$, $V_1 = \frac{q_1}{C_1}$, $V_2 = \frac{q_2}{C_2}$

$$V = V_1 + V_2$$

$\frac{\cancel{q}}{C} = \frac{\cancel{q}}{C_1} + \frac{\cancel{q}}{C_2} \rightarrow$

$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$	Series Capacitance

Reverse formulas from those for resistance!

Example.

A 1.50V cell is ~~in series~~ connected with $C_1 = 275\ \mu F$ and $C_2 = 38\ \mu F$. All series.

a) What value should a single replacement capacitor have?

$$\frac{1}{C} = \frac{1}{275\times10^{-6}} + \frac{1}{38\times10^{-6}}$$

$$\frac{1}{C} = 29952$$

$$C = 33.4\ \mu F$$

b) How much charge has the battery placed on each capacitor? What are their voltages?

$$\cancel{q = q_1 + q_2}$$

$$\cancel{q_1 =} \quad V = \frac{q}{C}$$

$$q = 1.50 \times 33.4\times10^{-6}$$

$$q_1 = q_2 = q = 50.1\times10^{-6}\,C$$

$$V_1 = \frac{50.1\times10^{-6}}{C_1} = \frac{50.1\times10^{-6}}{275\times10^{-6}}$$

$$V_1 = 0.182V$$

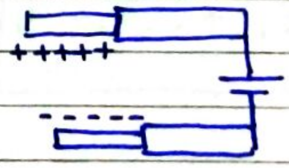
$$V_2 = \frac{50.1\times10^{-6}}{38\times10^{-6}} = 1.32V$$

In series, charge is same across all ~~resistors~~ capacitors & battery.

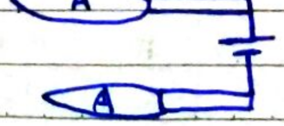
In parallel, voltage is same across each capacitor and battery.

This part is same like resistors.

4. Diaelectric Materials



- A simple capacitor has 2 conductors separated by non-conductive space.
- Thus, even wires in a circuit have capacitance, which is dangerous in high frequency circuits.
- Large capacity in a small space is premium. We use conductive plates rather than wires to store more charge. Capacitor is better with conductive plates.



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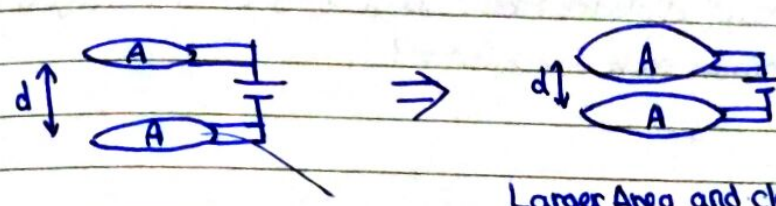
- The 2 ways to increase capacitance:

See formula for easy reference.

1) Increase the area over which charge can be stored. The A gets bigger.
2) Decrease distance between the plates.



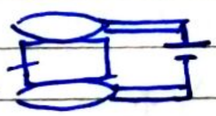
Larger Area and closer to each other (plates).

- Alright. Now another point: 3rd way to increase capacitance.

1) $F = \frac{kq_1 q_2}{d^2}$, we see that attractive force grows exponentially as d, plate separation, decreases.

2) Charge leakage can occur through air or vacuum. Hence, we ~~use~~ use a non-conductive material called a dielectric between the plates.

3) This reduces the electric force, increasing capacitance of capacitor.



$C = \frac{\varepsilon A}{d}$	Parallel-plate Capacitor

Kinda like $R = \frac{\rho L}{A}$

ε is permittivity of the dielectric.

Permittivity of free space is ε_0. This is $8.85 \times 10^{-12} \text{ Fm}^{-1}$. Farad per metre.

$\boxed{\varepsilon > \varepsilon_0}$

- What the dielectric does numerically: reduces electric field by $\frac{\varepsilon_0}{\varepsilon}$.

- The dielectric creates an oppositely-oriented electric field, which diminishes the overall E-field between the plates.

Date

EXAMPLE: A 275 µF capacitor will be manufactured using a dielectric having a permittivity of $4.00\,\varepsilon_0$ and circular plates having diameter 2.50 cm. What should the plate separation (& thickness of dielectric) be? Is it likely that this large a capacitor would be constructed using parallel plate architecture?

$$C = \frac{\varepsilon A}{d}$$

$$275 \times 10^{-6} = \frac{(4.00 \times 8.85 \times 10^{-12}) \times \pi (0.0125)^2}{d}$$

$$d = \frac{(4.00 \times 8.85 \times 10^{-12}) \times \pi (0.0125)^2}{275 \times 10^{-6}}$$

$$d = 6.32 \times 10^{-11}\ \text{m}$$

- "Given that the plate separation is of the order of an atomic diameter, it is very unlikely that one could construct a 275 µF parallel plate capacitor."

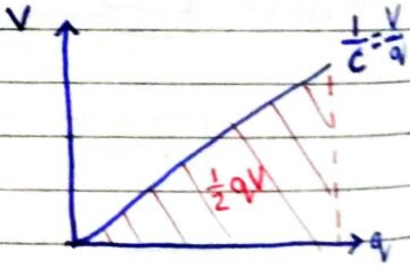
5) Energy Stored in a Capacitor

$$C = q/V$$

So, $V = \left(\frac{1}{C}\right) q$

$y = (m) x$

So, the graph of V vs. q has slope $\frac{1}{C}$.



Remember that Work done $= q\left(\frac{kQ}{r}\right) = qV$, when q & V are CONST.

Otherwise W = area under V vs. q graph.

As graph is a triangle, $W = \frac{1}{2} qV$ = Area under...

From conservation of energy, this is then the energy stored in a capacitor:

$E = \frac{1}{2} qV = \frac{1}{2} CV^2$	Energy stored in a capacitor

Date

PRACTICE: Find the energy stored in a $275\,\mu F$ capacitor charged up to $1.50\,V$.

$$E = \frac{1}{2}CV^2$$

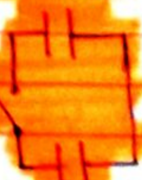
$$E = \frac{1}{2} \times 275 \times 10^{-6} \times 1.50^2$$

$$E = 3.09 \times 10^{-4}\,J$$

6) Solving problems involving parallel-plate capacitors

$C_1 = 275\,\mu F$

This is parallel.



$C_2 = 38\,\mu F$

Q. Two capacitors are shown. C_1 is fully charged at $1.50\,V$ and C_2 is initially uncharged.

a) What is the electrical energy stored in C_1?

$$E = \frac{1}{2}C_1V^2 = \frac{1}{2} \times 275 \times 10^{-6} \times 1.50^2$$

$$E = 3.09 \times 10^{-4}\,J$$

b) What is the charge on C_1's plates?

$E = \frac{1}{2}qV$ or $q = CV$

$3.09 \times 10^{-4} = \frac{1}{2}(1.5)q$ or $q = 275 \times 10^{-6} \times 1.5$

$q = 4.12 \times 10^{-4}\,C$ or $q = 4.13 \times 10^{-4}\,C$

c) When switch is closed, what charge will C_1 be? What charge will C_2 have?

In ~~Series~~ Parallel, $q = q_1 + q_2$, $V = V_1 = V_2$ ①

~~$q = 4.13 \times 10^{-4}\,C$~~

$C_{parallel} = C_1 + C_2 + \ldots$

Simple method to solve such discharge problems.
1) ~~Parallel or series??~~ Is V constant or q?
2) Equate the non-constant values with the constant. E.g. $V = \frac{C_1}{q_1} = \frac{C_2}{q_2}$ or $q = C_1V_1 = C_2V_2$
3) Using the equation, find ratio. $q_1 = \frac{C_1 q_2}{C_2}$ (substitution) $q_1 + q_2 = q_2 + \frac{C_1 q_2}{C_2} = q$

$$V = C_1\left(\frac{1}{q_1}\right) = C_2\left(\frac{1}{q_2}\right) \quad ②$$

$$q_2 = \frac{C_2}{C_1}q_1 \quad ③$$

$$4.13 \times 10^{-4} = q_1 + \frac{C_2}{C_1}q_1 = q_1\left(1 + \frac{38}{275}\right), \quad q_1 = \frac{4.13 \times 10^{-4}}{1 + 38/275} = 3.63 \times 10^{-4}\,C$$

$$q_2 = (4.13 \times 10^{-4}) - (3.63 \times 10^{-4}) = 0.50 \times 10^{-4}\,C$$

Good question. Circuit is actually parallel. Kinda makes sense. If it was series, the capacitors would not be drawn in opposing "parallel" wires.

EXAMPLE : A capacitor with no dielectric has capacitance 325 pF. It is charged up to 6.00V by momentarily attaching it to a battery, and then disconnecting it.

a) What is the energy stored in capacitor?

Ans. $E = \frac{1}{2}CV^2 = \frac{1}{2}(325\times10^{-12})(6^2) = 5.85\times10^{-9}$ J

b) What is the charge on capacitor?

Ans. $q = CV = 325\times10^{-12}\times6.00 = 1.95\times10^{-9}$ C

c) A dielectric with permittivity of $2\varepsilon_0$ is inserted. What is the new capacitance?

Ans. $C = 2\left(\frac{\varepsilon_0 A}{d}\right) = 2\times325\times10^{-12} = 650$ pF

d) What is the charge on the new capacitor?

Ans. $\cancel{q = 650\times10^{-12}\times6.00 = 3.90\times10^{-9}\text{ C}}$ Conservation of charge.
$q = 1.95\times10^{-9}$ C
Charge remains even if A, ε, d ~~are changed~~. Voltage changes which changes capacitance.

e) What is the voltage on new capacitor?

$V = \frac{q}{C} = \frac{1.95\times10^{-9}}{650\times10^{-12}} = 3V = 3.00V$

f) What is the energy stored in new capacitor?

Ans. $E = \frac{1}{2}CV^2 = \frac{1}{2}(650\times10^{-12})(3^2) = 2.92\times10^{-9}$ J

g) Explain discrepancy between (f) and (a).

Ans. Without dielectric, twice ~~about half~~ the energy is stored. Our target is to increase capacitance, not have higher energy output. ~~The dielectric cancels some of the electric force, and requires more work done~~.

Q) Define electric field strength.

The force per unit charge at any point in a space is electric field strength.

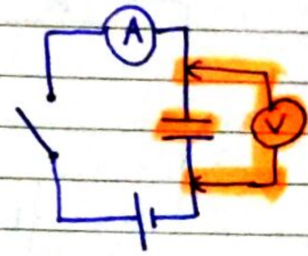
KIKY

Remember the graphs: V ↑ q ↑ I ↓ as the capacitor is charged.

• "Problems with charging are only graphic"

Date

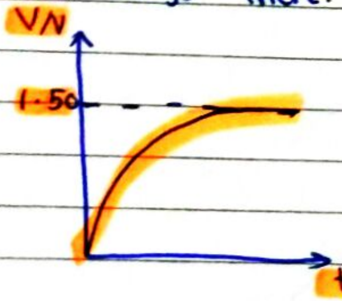
7) Charging a Capacitor = Pretty important



PRACTICE: 1.50V cell is connected to a 275 μF capacitor in the circuit shown.

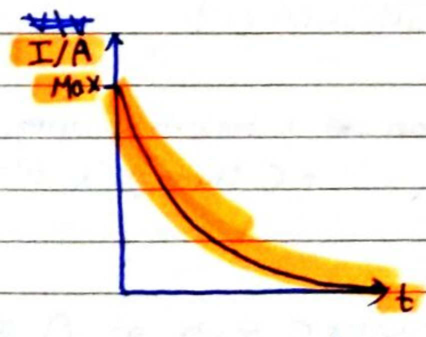
a) Make a sketch graph of what the voltmeter reads once switch is closed.

Sol. it is 0V first and then increase to 1.50V, and then stays there.



b) Sketch what the ammeter reads once switch is closed.

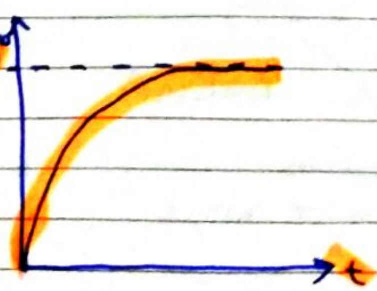
Sol. • It is initially 0.
• After switch closes, it begins at a maximum and then decreases rapidly as plates "fill up" with charge & repel further charge.
• Then it becomes 0.



c) Sketch what charge q on plates will be once switch is closed.

q/C

• Increases from 0 to a maximum. $q = CV = 4.1 \times 10^{-4}$
• Rapid and then slows down due to repulsion.
• Charge reaches a maximum value on the plates, stopping the flow of further charge.



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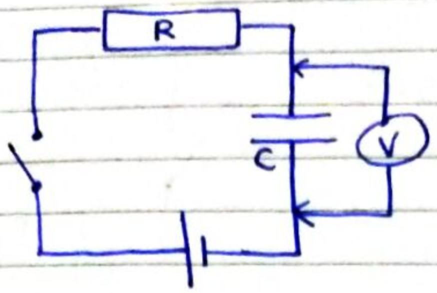
Graphs!

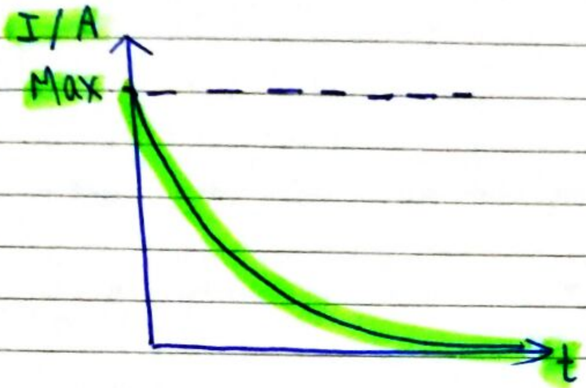
Date

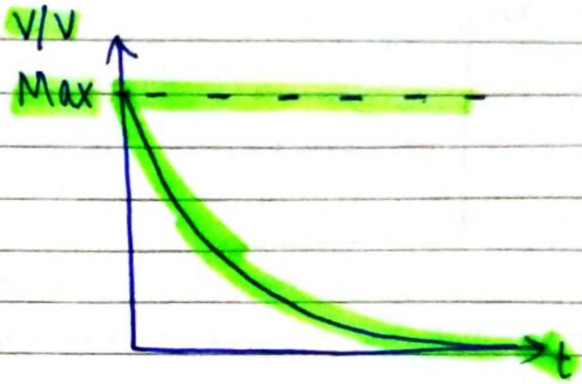
8) RC series circuits - charging

RC = circuit with both capacitor & resistor



Make a sketch showing voltage & current across resistor after the switch is closed as RC increases.

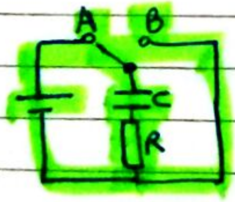




Current decreases rapidly as plates fill up with charge & repel further charge. Then, I=0A

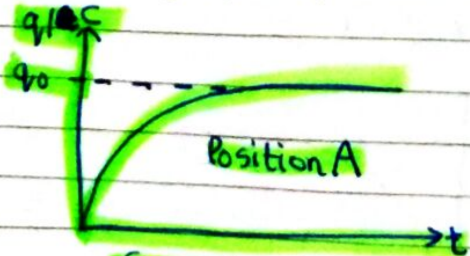
Voltage across resistor is ∝ to I, Same shape graph. As charge is stored in capacitor, V across the resistor rapidly decreases.

9) RC series circuits - discharging IMPORTANT!

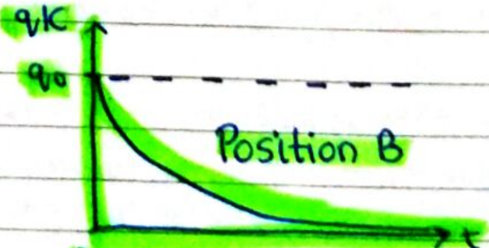


A two-position switch can alternately charge (A) and discharge (B) a capacitor C through a resistor R.

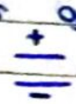
Sketch graphs of what charge across capacitor C reads at A and B.



Gets charged

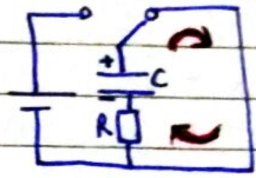


Capacitor discharges through wire & the resistor.

- First, at A, capacitor gets charged. There is P.D. across the plates of the capacitor. 

- At B, the (+) on top plate travels through wire and ~~capac~~ resistor to reach bottom plate and reduced potential difference upto 0.

- Using Kirchhoff's rule for V. See the loop.



$$\Sigma V_i = 0 \rightarrow V_C = IR$$

$V_C = \frac{q}{C}$, $I = \frac{\Delta q}{\Delta t}$, so we have

$$\frac{q}{C} = R\left(\frac{\Delta q}{\Delta t}\right)$$

$$\frac{q}{CR} = \frac{\Delta q}{\Delta t}$$

Δq is always negative during discharge.

$$\frac{q}{CR} = \frac{-\Delta q}{\Delta t}$$

$I = \frac{\Delta q}{\Delta t} = \frac{-q}{\tau}$ For discharge, Must remember τ = time constant	**RC Discharge**
$q = q_0 e^{-\frac{t}{\tau}}$ where $\tau = RC$ (seconds)	$\tau = RC$ = time constant (seconds)

The 2nd equation in derived through calculus from the first. But, proof it beyond syllabus. It turns out that $q = q_0 e^{-t/\tau}$ is the first equation's time-dependent solution.

EXAMPLE: A 275 μF capacitor is charged to 6.00V. Discharged through a 10.0 MΩ resistor.

a) Find time constant.

$\tau = RC$

$\tau = 10\times10^6 \times 275\times10^{-6} = 2750\text{s}$

b) Find initial charge on plates.

$q = CV$

$q = 275\times10^{-6}\times 6 = ~~1650~~ ~~\mu A~~ 1650\,\mu C$.

c) Find charge on plates exactly one time constant after discharge has begun.

$q = q_0 e^{-\frac{t}{\tau}} = q_0 e^{-\frac{\tau}{\tau}}$

$q = 1650\times10^{-6} \times \frac{1}{e}$

$q = 6.07\times10^{-4}$ C

Date

Because of the ~~net~~ relationships $V = \frac{q}{C}$ and $I = \frac{V}{R}$, we can easily derive the next 2 formulas:

Instantaneous not average Look at questions below.

$I = I_0 e^{-\frac{t}{\tau}}$	where $I_0 = \frac{-q_0}{RC}$ Must remember	RC
$V = V_0 e^{-\frac{t}{\tau}}$	and $V_0 = \frac{q_0}{C}$ Must remember	Discharge

$V = V_0 e^{-t/\tau}$ is voltage across capacitor, NOT RESISTOR!

Kirchoff's rule for V would produce resistor voltage formula as $V_R = V_0(1 - e^{-t/\tau})$

EXAMPLE: A 275μF capacitor is charged to 6.00V. Then discharged through a 10.0 MΩ resistor. Find average current during the first 1375 s of discharge.

~~a) Find time constant.~~

~~$\tau = RC$~~

~~$\tau = 10 \times 10^{6} \times 275 \times 10^{-6} = 2750$~~

~~$\tau = RC$~~

~~$\tau = 10 \times 275 = 2750 s$~~

$\tau = RC = 2750 s$

$q_0 = CV = 275 \times 10^{-6} \times 6$

$q_0 = 1650 \times 10^{-6} C$

~~$I = I_0 e^{-\frac{1375}{2750}}$~~

~~$I_0 = \frac{q_0}{RC}$~~

$I = \frac{\Delta q}{\Delta t}$

~~$I_0 = \frac{CV}{RC} = \frac{V}{R} = \frac{6}{10 \times 10^6} = 0.6 \times 10^{-6} A$~~

$\Delta q = (0.00165 e^{-\frac{1375}{2750}} - 0.00165 e^{-0})/1375$

I ~~Δq~~ $= \frac{0.001 - 0.00165}{1375}$

~~$I = 0.6 \times 10^{-6} \times e^{-\frac{1}{2}}$~~

~~$I = 3.64 \times 10^{-7} A$~~

~~Δq~~ $I = \boxed{-}4.73 \times 10^{-7} A$

You may ignore it.

~~Exa~~

EXAMPLE: 275 μF capacitor is charged to 6.00V. Then discharged through a 10 MΩ resistor. Find instantaneous current at $t = 1375s$.

$I = I_0 e^{-\frac{t}{\tau}}$

~~We cannot do $\frac{R}{V}$ because it is not about resistor, but capacitor instead.~~

$q = q_0 e^{-t/\tau}$

$q =$ ~~275×10⁻⁶~~ $CV e^{-\frac{1}{2}}$

$q = \frac{275 \times 10^{-6} \times 6}{\sqrt{e}} = 1.00 \times 10^{-3} C$

OR

$I = -\frac{q}{RC}$

$I = \frac{1.00 \times 10^{-3}}{2750}$

$I = \boxed{-}3.64 \times 10^{-7} A$

You may ignore it.

KIKY

PRACTICE

Date

A 275 μF capacitor is charged up to 6.00V. Then discharged through a 10.0 $M\Omega$ resistor. Find instantaneous voltage at t = 1375s. Find instantaneous current through Ohm's law.

$$q = CVe^{-\frac{1}{2}}$$

Voltage in capacitor, not resistor

$$q = \frac{275\times10^{-6}\times6}{\sqrt{e}} = 0.001$$

$$\cancel{V_0 = \frac{q_0}{C} = 275\times}$$

$$V = \frac{q}{C} = \frac{0.001}{275\times10^{-6}} = 3.64\text{ V}$$

$$V = V_0 e^{-\frac{1}{2}}$$

$$V = \frac{6}{\sqrt{e}}$$

$$= 3.64\text{V}$$

$$I = \frac{V}{R} = \frac{3.64}{10\times10^{6}} = 3.64\times10^{-7}\text{ A}$$

- Half-life of Capacitor : When its voltage/charge will have decayed to half its original value.

$$\frac{1}{2}V_0 = V_0 e^{-\frac{T_{1/2}}{\tau}}$$

$$\frac{1}{2} = e^{-\frac{T_{1/2}}{\tau}}$$

$$\ln\frac{1}{2} = -\frac{T_{1/2}}{\tau}$$

$$-\ln\frac{1}{2} = \frac{T_{1/2}}{\tau}$$

$$\ln 2 = \frac{T_{1/2}}{\tau}$$

$T_{1/2} = \tau \ln 2$	Half-Life of a capacitor

Remember it

RACTICE: Timer using capacitor & resistor needs RC circuit to have halflife 2500 seconds. It has 275 μF capacitor. What value should resistor have?

$$T_{1/2} = RC\ln 2 \;;\; R = \frac{2500}{275\times10^{-6}\times\ln 2} = 1.31\times10^{7}\,\Omega = 13.1\,M\Omega$$

KIKY