Topic 9 : Wave Phenomena AHL

9.1 Simple Harmonic Motion

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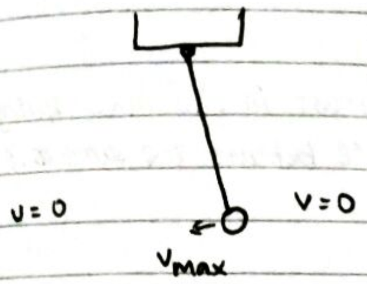
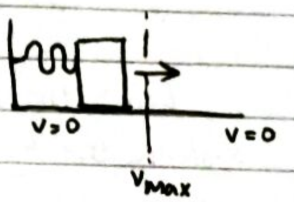
1) Examples of oscillation

Oscillations are repeating vibrations.





$f = \frac{1}{T}$	Relation with T & f.

Frequency f - oscillations per second
Period T - seconds per oscillations

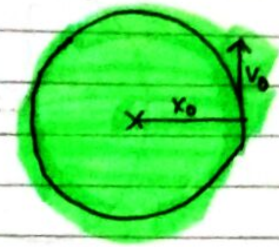
2) Angular speed ω

Let's revisit UCM- Uniform Circular Motion
(I know, it's circles, not waves)

There is constant speed v_0 and radius x_0

$a = \frac{v_0^2}{x_0}$

$a_c = \frac{v^2}{r}$



If T is 12 sec.

$\omega = \frac{360°}{12} = 30°$ or $\boxed{\frac{\pi}{6} \text{ radians}}$
Use this!

Angular speed is measured in Radians per second.

$\omega = \frac{2\pi}{T} = \frac{\theta}{t} = 2\pi f$	Angular speed or angular frequency

Example. Find the angular frequency of the second hand of a clock.

T = 60 seconds

$\omega = \frac{2\pi}{60} = \frac{\pi}{30} = 0.105 \text{ rad s}^{-1}$

ii) For rotation of Earth!
T = 86400 s

$\omega = \frac{2\pi}{86400} = 7.3 \times 10^{-5} \text{ rad s}^{-1}$
That's why we can't feel it spinning.

An object travels at speed v_0 in a circle of radius x_0. Period is T.

a) Speed in terms of x_0 and T.

$v_0 = \frac{2\pi x_0}{T}$

b) Show that $v_0 = x_0\omega$

$v_0 = \left(\frac{2\pi}{T}\right)x_0$

$v_0 = x_0\omega$

$v = \omega r$

c) Find centripetal acc. a_c in x_0 and ω

$a_c = \frac{v_0^2}{x_0}$

$a_c = \frac{(x_0\omega)^2}{x_0}$

$a_c = \frac{x_0\omega^2}{1} = x_0\omega^2$

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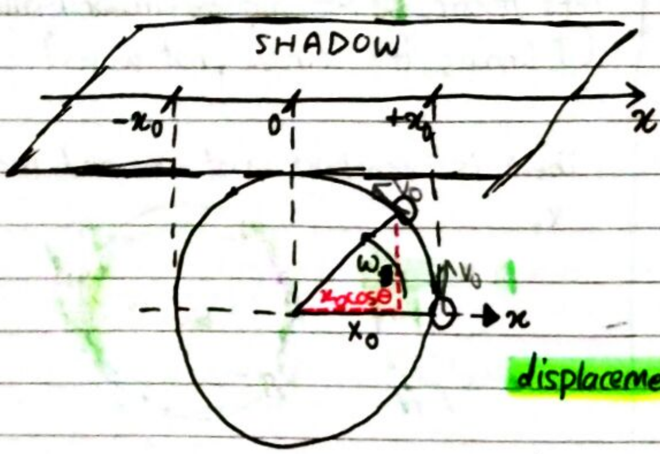
3) Back to SHM! The defining equation of SHM is $a = -\omega^2 x$

How does an oscillating spring relate to UCM?



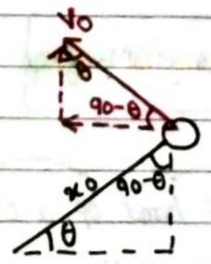
Like the mass in the mass-spring system, the ball behaves the same as it moves.

The shadow's displacement or horizontal displacement is $\boxed{x = x_0 \cos\theta}$

Since $\omega = \frac{\theta}{t}$, $\theta = \omega t$

Now, $\boxed{x = x_0 \cos \omega t}$

Now, read carefully. The red triangle is taken down...

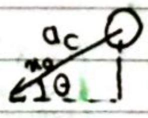


We can get the x-component of velocity. we use sign.

$v = -v_0 \sin\theta$

$\theta = \omega t$ and $v_0 = x_0 \omega$

$\boxed{v = -x_0 \omega \sin \omega t}$



(Negative because it point ~~towards center, which is current or~~ to the left.)

$a = -a_c \cos\theta$

$\theta = \omega t$ $\quad a_c = x_0 \omega^2$

$a = -x_0 \omega^2 \cos \omega t$

Since $x = x_0 \cos \omega t$, $a = -\omega^2 x$

$x = x$	$x = x_0 \cos \omega t \, (x_0 \sin \omega t)$	$\omega = \frac{2\pi}{T}$
$v = -x_0\omega$	$v = -x_0 \omega \sin \omega t \, (x_0 \omega \cos \omega t)$	$\omega = 2\pi f$
$a = -x_0\omega^2$	$a = -x_0 \omega^2 \cos \omega t \, (-x_0 \omega^2 \sin \omega t)$	$v_0 = x_0 \omega$
	$a = -\omega^2 x$	

Equations of simple harmonic motion

The trig. is only to find x using x_0 & radian angle.

x_0 = max. displacement

v_0 = max. speed

Just differentiate!

These work when mass begins at $x = +x_0$ at $t = 0s$. Like a cos graph if talking about displacement. (When mass begins at $x = 0$ at $t = 0s$.)

Data booklet has highlighted formulas.

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Now $x = x_0 \cos\omega t$, $v = -v_0 \sin\omega t$ and $v_0 = x_0\omega$

Square each side:

$$x^2 = x_0^2 \cos^2\omega t$$
$$v^2 = (-x_0\omega \sin\omega t)^2 = x_0^2\omega^2 \sin^2\omega t$$

$\sin^2\omega t + \cos^2\omega t = 1$, Identity!

So,

$$v^2 = x_0^2\omega^2(1 - \cos^2\omega t)$$
$$v^2 = \omega^2(x_0^2 - x_0^2\cos^2\omega t)$$

Remember: $x^2 = x_0^2\cos^2\omega t$

$v^2 = \omega^2(x_0^2 - x^2)$, which becomes

$v = \pm\omega\sqrt{x_0^2 - x^2}$	Relation between x & v.

$F = -kx$

$5g = -kx$

$g = -\left(\frac{k}{m}\right)x$

4) Solving SHM problems

A spring has spring constant $125\ nm^{-1}$, is attached to 5.0kg mass, Stretched +4.0m, then released from rest.

a) Using Hooke's Law, show that mass-spring undergoes SHM with $\omega^2 = \frac{k}{m}$

$F = -kx$

$F = ma$

$ma = -kx$

$a = \left(\frac{-k}{m}\right)x$

$a = -\omega^2 x$

So, $\omega^2 = \frac{k}{m}$

Therefore, undergoes SHM

b) Angular frequency, frequency and period of oscillation.

$\omega^2 = \frac{k}{m}$

$\omega = \sqrt{\frac{125}{5}}$

$\omega = 5\ rad\,s^{-1}$

$\omega = \frac{2\pi}{T}$

$5T = 2\pi$

$T = \frac{2\pi}{5}$

$T = 1.26\ sec$

$= 1.3\ sec$

$f = \frac{5}{2\pi}$

$f = 0.80\ Hz$

c) Show that the position and velocity of the mass is given by $x = 4\cos 5t$ and that $v = -20\sin 5t$.

Since it starts from max. displacement,

$x = x_0 \sin\omega t$ $\quad x_0 = 4 \quad \omega = 5$

$x = 4\cos 5t$

$v = -\omega x_0 \sin\omega t$

$v = -20\sin 5t$

d) Find position, velocity, acceleration of mass at $t = 0.75s$. Then find maximum KE.

$x = 4\cos 5(0.75) = -3.3m$, $v = 11.4\ ms^{-1}$, $a = -\omega^2 x_0 \cos\omega t = -100\cos(5)(0.75) = 82\ ms^{-2}$

$a = -\omega^2 x \quad V_{max} = -$

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c) Derive formula for position and velocity.

$x = x_0 \cos \omega t$ $\qquad$ $v = -\omega x_0 \sin \omega t$

$x_0 = 4m \quad \omega = 5\ rad\,s^{-1}$ $\qquad$ $v = -20 \sin 5t$

$x = 4 \cos 5t$

d) Find the position, velocity, acceleration at $t = 0.75s$. Find max. kinetic energy.

$x = 4 \cos 5(0.75)$ $\qquad$ $v = -20 \sin 5t$ $\qquad$ $a = -\omega^2 x$

$= -3.3m$ $\qquad$ $v = 11 ms^{-1}$ $\qquad$ $a = -25(-3.3)$

$= 83 ms^{-2}$

$v_0 = -\omega x_0 \ (\sin\theta = 1)$

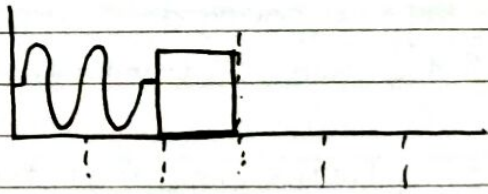
$v_0 = -5 \times 4 = -20 ms^{-1}$

$KE = \frac{1}{2}mv^2 = \frac{1}{2} \times 5 \times 400 = 1000 J$

5) The period of a mass-spring system

• Since $\omega^2 = \frac{k}{m}$ for a mass-spring system and $\omega = \frac{2\pi}{T}$ for any system,

we can write



$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\sqrt{\frac{k}{m}}}$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$T = 2\pi\sqrt{\frac{m}{k}}$	Period of a mass-spring system

Practice: Find period of 25 kg mass placed in oscillation on end of a spring having $k = 150 Nm^{-1}$.

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{25}{150}} = 2\pi\sqrt{\frac{1}{6}} = 2.56 \approx 2.6 \text{ sec}$$

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6) The period of a simple pendulum

For a simple pendulum of length L,

$$\omega = \sqrt{\frac{g}{L}} \quad \text{(without proof)}$$

Then, $T = \dfrac{2\pi}{\omega}$

$= \dfrac{2\pi}{\sqrt{\frac{g}{L}}}$

$T = 2\pi\sqrt{\dfrac{L}{g}}$

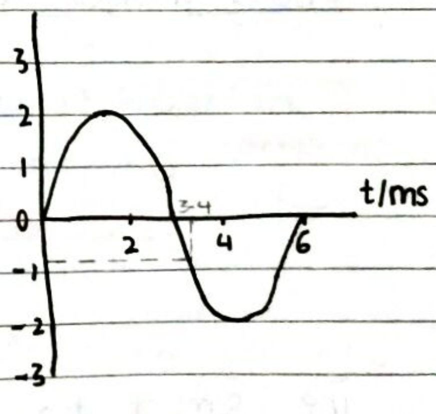
$T = 2\pi\sqrt{\dfrac{L}{g}}$	Period of a simple pendulum

Practice. Find period of 25 kg mass placed in oscillation on the end of a 1.75 m string.

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.75}{\cancel{25}\ 9.8}} = 2.7\,\text{sec}$$

7) Solving problems graphically and by calculation

a) Period, angular velocity and frequency?



Period = 6 ms = 0.006 s = 6×10^{-3} s

$\omega = \dfrac{2\pi}{6} = 1.05\ \text{rad ms}^{-1} = \cancel{0.001}\ 1050\ \text{rad s}^{-1}$

$f = \dfrac{1}{6\times10^{-3}} = 170\ \text{Hz}$

b) Velocity and acceleration at t = 3.4 s

$v = \pm\,\omega\sqrt{x_0^2 - x^2}$

$v = \pm\,1050\sqrt{0.002^2 - \cancel{(-0.8)}(-0.0008)^2}$

$v = -1.89\ \text{ms}^{-1}$ (negative because slope is negative)

$a = -\omega^2 x = -(1050)^2 \times (-0.0008)$

$a = 882\ \text{ms}^{-2} = 880\ \text{ms}^{-2}$

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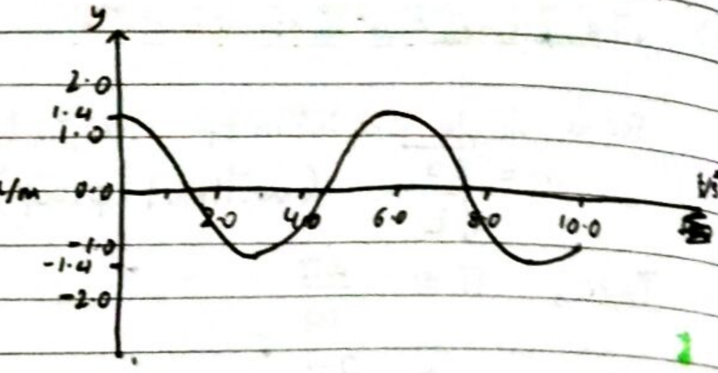
(i) Displacement-time graph for 2.5kg mass is shown.



a) Period, ω and v_{max}.

$T = 6$ sec

$\omega = \frac{2\pi}{6} = 1.04 = 1.0$ rad s^{-1}

$v_{max} = 1.047 \times 1.4 = 1.5 ms^{-1}$ $\qquad$ $v_{max} = \omega x_0$

b) Find force on the mass at $t = 3s$. Then find its velocity at that instant.

$F = ma$

$a = -\omega^2 x$

$x = -1.4$

$a = -1.04^2 \times -1.4$

$a = 1.5 ms^{-2}$

$F = 1.5 \times 2.5 = 3.8N$

$v = \pm \omega \sqrt{x_0^2 - x^2}$

$v = \pm 1.04 \sqrt{1.4^2 - (1.4)^2}$

$v = \pm 1.04 \sqrt{0}$

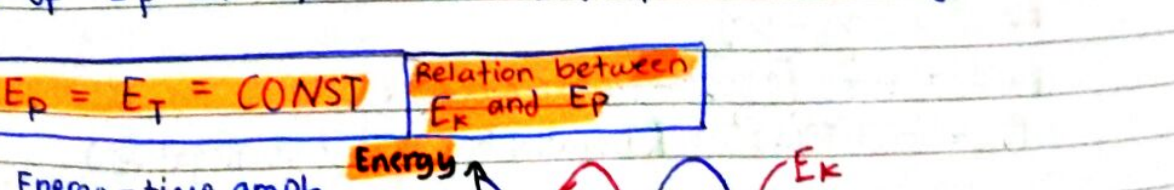
$v = 0 ms^{-1}$

Haha, makes sense!

So far, everything is understandable if you read properly!

8) Energy changes during SHM

A continuous exchange between potential energy and kinetic energy occurs in SHM.



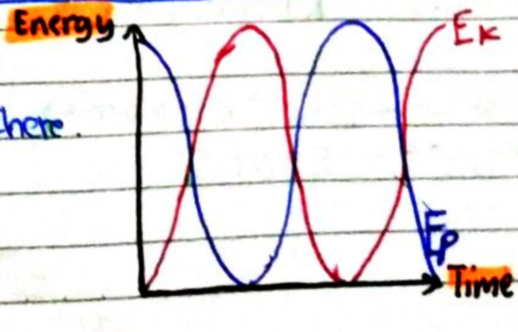
The sum of E_p and E_K is constant, if friction and drag are both 0.

$\boxed{E_K + E_p = E_T = CONST}$ Relation between E_K and E_p

This is the Energy-time graph.

The SHM starts with max PE here.

~~It can start with~~



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Remember: $v = \pm\omega\sqrt{(x_0^2 - x^2)}$

Then,
$$v^2 = \omega^2(x_0^2 - x^2)$$
$$\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

$$KE = \frac{1}{2}m\omega^2(x_0^2 - x^2)$$

$E_K = \frac{1}{2}m\omega^2(x_0^2 - x^2)$	Relation between E_K and x

In data booklet

Remember $v_{Max} = v_0 = \omega x_0$

So,
$$E_{K_{max}} = \frac{1}{2}m\omega^2(x_0^2)$$

$E_T = E_{K_{max}} = \frac{1}{2}m\omega^2 x_0^2$	Relation $E_{K_{max}}$ and x_0 $E_{K_{max}} = E_T$

In data booklet.

Basically $E_{K_{max}}$ is Total energy because E_p is 0 when E_K is max.

Example. A 3.00 kg mass undergoes SHM with period 6.00 s. Amplitude is 4.00 m.

a) Maximum kinetic energy and what is x when this occurs?

$$E_{K_{max}} = \frac{1}{2}m\omega^2 x_0^2$$

$$\omega = \frac{2\pi}{6}$$
$$\omega = \frac{\pi}{3} \text{ rad s}^{-1}$$
$$E_{K_{max}} = \frac{1}{2}(3.00)\left(\frac{\pi}{3}\right)^2(4)^2$$
$= 26.3$ J, when $x = 0$

b) What is E_p when E_K is max. and what is the total energy of the system?

$E_p = 0$ J

$E_T = 26.3$ J

c) What is E_p when kinetic energy is 15.0 J?

$E_p = E_T - E_K$

$= 26.3 - 15.0$

$= 11.3$ J

$E_{K_{max}} = E_T$

$E_T = \frac{1}{2}m\omega^2 x_0^2$	Relation between E_T and x_0

Date

$E_k = \frac{1}{2}m\omega^2(x_0^2 - x^2)$

$E_K = \frac{1}{2}m\omega^2 x_0^2 - \frac{1}{2}m\omega^2 x^2$

$E_k = E_T - \boxed{\frac{1}{2}m\omega^2 x^2}$ - must be E_p

$E_p = \frac{1}{2}m\omega^2 x^2 = \frac{1}{2}kx_0^2$	Potential energy E_p

Not in data booklet!

Example

3.00 kg mass undergoes SHM with period 6.00 s. Amplitude is 4.00m. Find ~~E_k~~ E_p when $x = 2.00$ m. Find E_K at $x = 2.00$ m.

$M = 3$ kg
$T = 6$
$x_0 = 4$

$E_p = \frac{1}{2}m\omega^2 x^2$

$= \frac{1}{2}(3)(\frac{2\pi}{6})^2(2)^2$

$= (1.5)(4)(\frac{4\pi^2}{36})$

$= \frac{1}{6} \times 4\pi^2$

$= \frac{2}{3}\pi^2 = 6.58$ J

$E_p = \frac{1}{2}(3)(\frac{2\pi}{6})^2(2^2)$

$E_p = 6.58$ J

$E_K = \frac{1}{2}(3)(\frac{2\pi}{6})^2(4^2 - 2^2)$

$E_K = 19.7$ J

$E_K = \cancel{6.58} \; \frac{1}{2}(3)(\frac{2\pi}{6})^3(16) - 6.58$

$= 24(\frac{\pi^2}{9})$

$= 26.3$ J $- 6.58$

$= 19.7$ J

Practice. A 2.00 kg mass undergoes SHM with period 1.75 s.

a) What is total energy of system? $x_0 = 3$ m.

$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.75} = 3.59$ rad s^{-1}

$E_T = \frac{1}{2}m\omega^2 x_0^2$

$= \frac{1}{2}(2)(3.59)^2(3)^2 = 116$ J

a) $E = \frac{1}{2}(2)(\frac{2\pi}{1.75})^2(9)$

$E = 116$ J

b) What is E_p when $x = 2.50$ m?

$E_p = \frac{1}{2}m\omega^2 x^2$

$= \frac{1}{2}(2)(3.59)^2(2.5)^2 = 80.6$ J

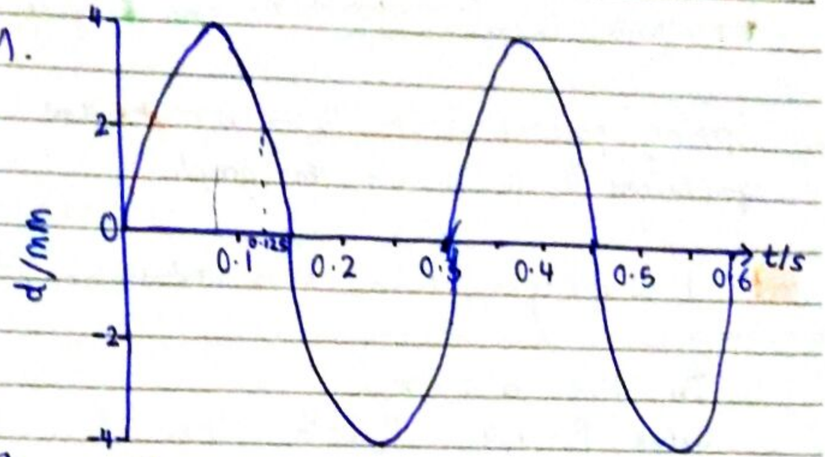
$E_p = \frac{1}{2}(2)(\frac{2\pi}{1.75})^2(2.5^2)$

$E_p = 80.6$ J

(KKY)

Practice.

2 kg mass undergoes SHM.



a) Total energy of the system?

$E_T = \frac{1}{2} m \omega^2 x_0^2$

~~$\frac{1}{2}(2)($~~

$T = 0.3s$

$\omega = \frac{2\pi}{0.3} \text{ rad } s^{-1}$

$x_0 = 0.4cm = 0.004m$

$E_T = \frac{1}{2}(2)(\frac{2\pi}{0.3})^2(0.004)^2$

$= 0.007 J$

b) P_E at $t = 0.125s$?

$x = 0.002 m$

$P_E = \frac{1}{2} m \omega^2 x^2$

$= \frac{1}{2}(2)(\frac{2\pi}{0.3})^2(0.002)^2$

$= 0.002 J$

c) E_K at $t = 0.125s$

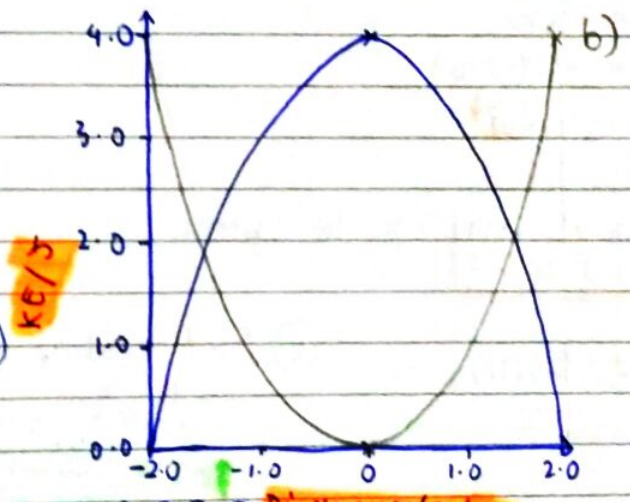
$E_k = E_T - E_p$

$E_k = 0.007 - 0.002$

$E_k = 0.005 J$

9) Sketching and determining graphs of SHM

Q. Kinetic energy vs. displacement for a system undergoing SHM is shown. 0.125 kg mass on a spring.



a) Determine max. velocity of the mass.

$KE = 4 = \frac{1}{2} m v^2$

$\frac{8}{0.125} = v^2 = 64$

$v = \cancel{8 ms^{-1}}\ 8.0 ms^{-1}$

b) Sketch E_p and determine E_T in system.

$E_T = 4.0 J$

c) Determine spring constant k of the spring.

$E_p = \frac{1}{2} k x_0^2$

$\frac{1}{2} k x_0^2 = 4 J$

$\frac{1}{2} k (0.02^2) = \cancel{4J}\ 4J$

~~$k_0 = 2 Nm^{-1}$~~

$(k)\ 0.0004 = 8 J$

$k = 20000 Nm^{-1}$

d) Determine acc. at $x = 1.0 cm$ of the mass.

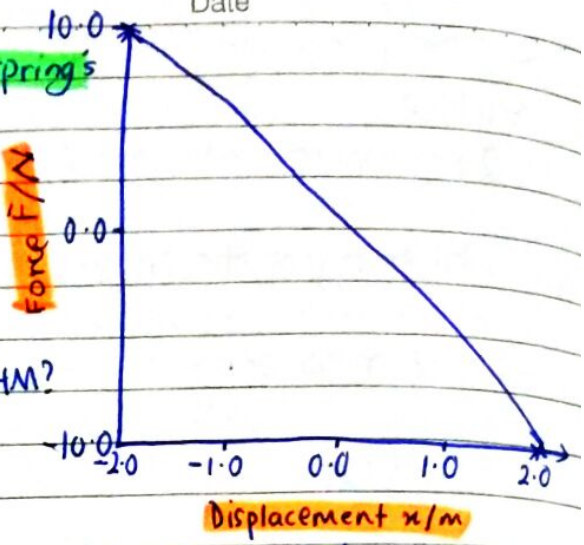
$F = -kx = ma$

$20000 \times 0.01 = 0.125 a$

$\frac{200}{0.125} = a$

$a = 1600 ms^{-2}$

Example. 4.0 kg mass is placed on a spring's end and displaced 2.0 m to the right.



Spring force F vs its displacement x from equilibrium is shown on the graph.

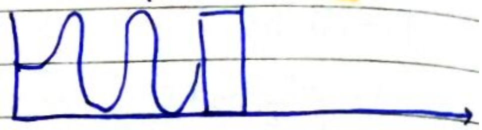
a) How do you know mass undergoes SHM?

In SHM $a \propto -x$

Since $F = ma$, $F \propto -x$ also.

The graph shows $F \propto -x$,

Thus it is in SHM.



b) Find spring constant of the spring.

When $x = -2.0$

$F = -kx$

$10 = -k(-2.0)$

$k = 5\ Nm^{-1}$

c) Find total energy of the system.

$E_T = \frac{1}{2} k x_0^2$

$= \frac{1}{2}(5)(4)$

$= 10\ J$

d) Find Max. speed of mass.

$\frac{1}{2} m v_{max}^2 = E_T$

$\frac{1}{2} m v_{max}^2 = 10\ J$

$m v_{max}^2 = 20$

$v_{max}^2 = 5$

$v = \sqrt{5} = 2.2\ ms^{-1}$

e) Find speed of mass when displacement is 1.0 m.

$E_T = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$10 = \frac{1}{2} m v^2 + 2.5(1)$

$7.5 = \frac{1}{2}(4) v^2$

$3.75 = v^2$

$v = 1.9\ ms^{-1}$

Topic 9.2 Single-slit diffraction

All this is for single-slit

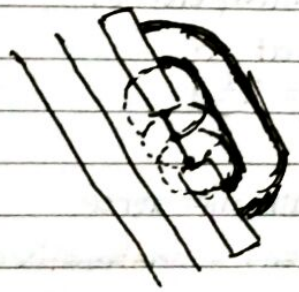
1) Diffraction through a single-slit and around objects

If the aperture or hole is much larger than the wavelength, diffraction is minimal.

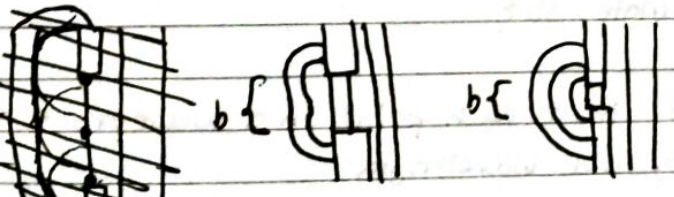
Diffraction happens within the medium.

Diffraction is the spreading out of waves by passing through a narrow aperture or across an edge.

Huygen's principle: Every point on a wavefront emits a spherical wavelet of the same velocity and wavelength as the original wave.
This is why waves turn corners.



The smaller the aperture, the more pronounced the diffraction effect.



$b = 6\lambda$ — Bigger aperture

$b = 2\lambda$ — Smaller aperture

The aperture size must be in order of the wavelength.

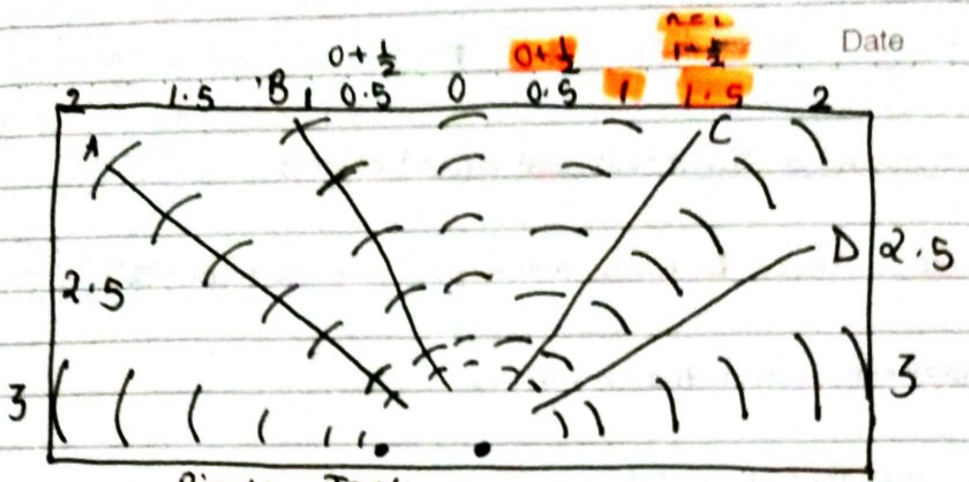
2) Path difference

Coherent means in phase and same frequency.

Path difference = $n\lambda$ n is an integer	Condition for constructive interference
Path difference = $(n+\frac{1}{2})\lambda$ n is an integer	condition for destructive interference

Constructive

Destructive



Ripple Tank

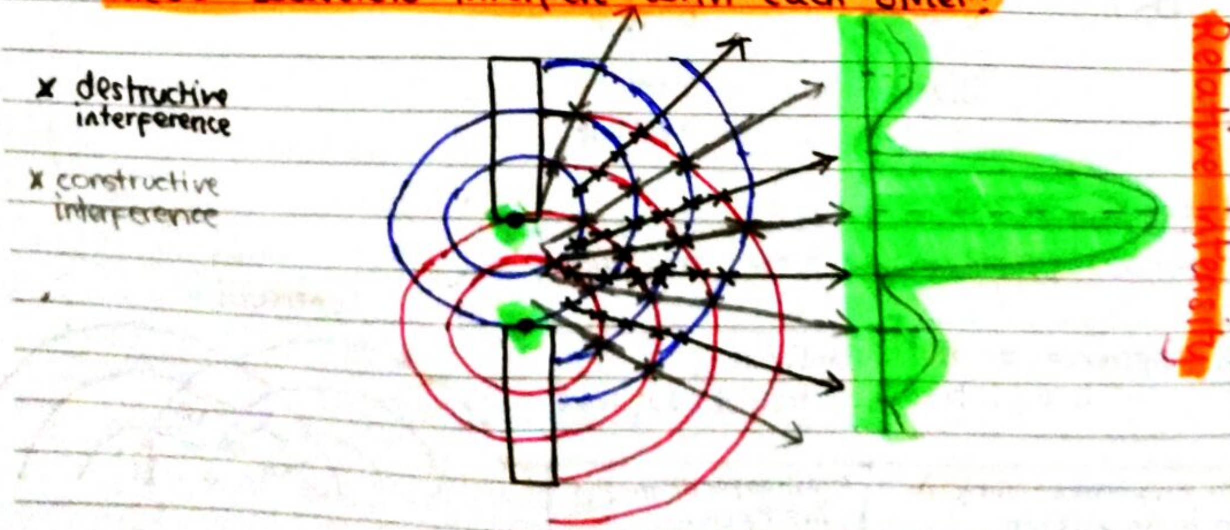
A and B are constructive interference.
$n = 1$ in B and $n = 2$ in A
$n\lambda$ = Path difference.

C and D are destructive interference.
$n = 1$ at C and $n = 2$ at D.
$1 + \frac{1}{2} = 1\frac{1}{2}\lambda$ and $2 + \frac{1}{2} = 2\frac{1}{2}\lambda$

1.5λ and 2.5λ are path difference.
minimums
So, at first, $n = 0$ ~~for one constructive and 2 destructive here.~~
maximum

3) <u>Diffraction through a single slit</u>

Huygen's principle states that each point on a wavefront emits a spherical wavelet of the same velocity and wavelength.
These wavelets interfere with each other!



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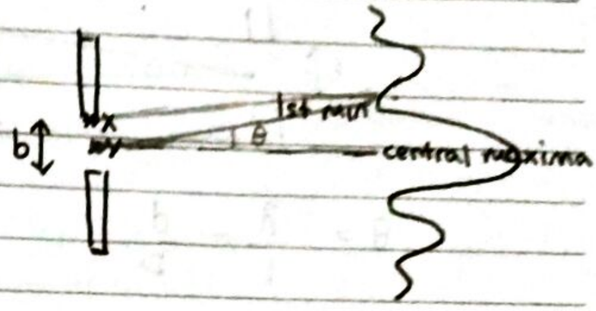
Formula derivation

Consider slit of width b.



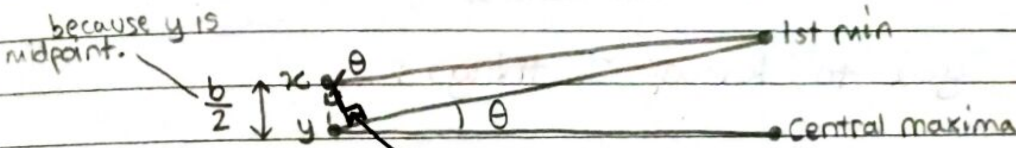
At the central maximum, all wavelets travel same distance, so constructive interference. No path difference.

Consider slit points X at one edge and y at the center:

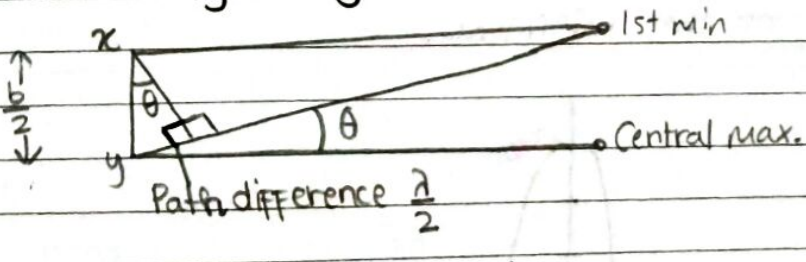
At 1st minimum, path difference is $\frac{\lambda}{2}$. Destructive interference.

y is the reference line. θ is angle between reference and 1st minimum.

Looks like this:



Construct a right angle.

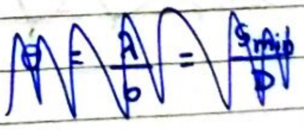


We see that $\sin\theta = \dfrac{\frac{\lambda}{2}}{\frac{b}{2}}$

$$\sin\theta = \frac{\lambda}{b}$$

Recall that θ is very small (in rads), so $\sin\theta \approx \theta$ (in rad)

$\theta = \dfrac{\lambda}{b}$	Location of first minimum in single slit diffraction



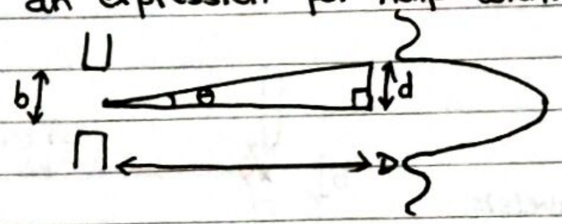
First min
$\theta = \frac{\lambda}{b} = \frac{\frac{s}{2}}{D} = \frac{s}{2D}$

Q a) Explain qualitatively, the intensity distribution in single-slit diffraction.

Ans. It is caused by path length difference and interference. When PLD is 0, there is central maxima, constructive interference. When PLD is $\lambda, 2\lambda, 3\lambda$, etc, these are smaller maximas caused constructively. 1st minima is at PLD $\frac{\lambda}{2}$. 2nd min is $\frac{3\lambda}{2}$. This is destructive interference.

Date

b) Derive an expression for half-width of central maxima.



$$\theta = \frac{\lambda}{b} = \frac{d}{D}$$

$$\frac{\lambda}{b} = \frac{d}{D}$$

$\frac{1}{2}s = d = \frac{\lambda D}{b}$ $\qquad s = \frac{2\lambda D}{b}$ $\qquad \frac{\lambda}{b} = \frac{\frac{s}{2}}{D}$ $\qquad s = \frac{2\lambda D}{b}$

~~The formula is $s = \frac{\lambda D}{d}$ in formula booklet. Same thing.~~

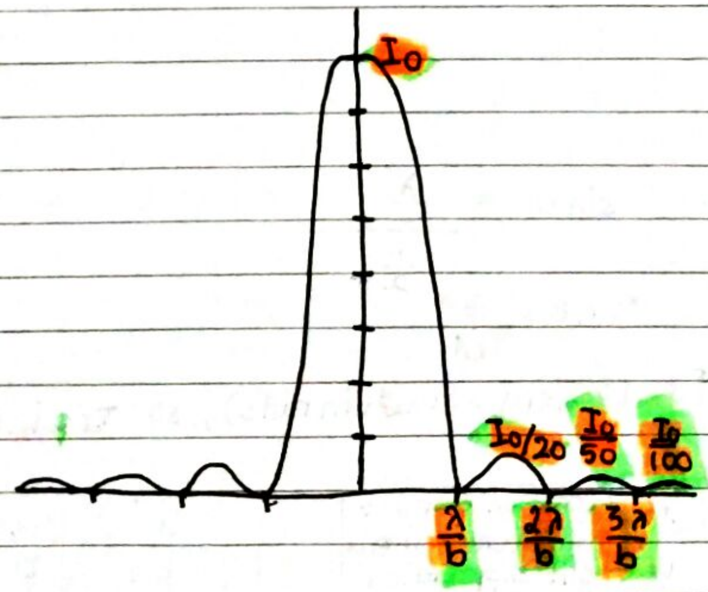
Remember, in $s = \frac{\lambda D}{d}$, s is distance between central & first maximas. Here, we aren't talking about this. This formula is Young's double slit experiment.

4) Intensity of a single-slit diffraction pattern

IBO expects you to know 2 things:

- the relative intensities of the maximums.
- the positioning of the minimums.

Here it is:

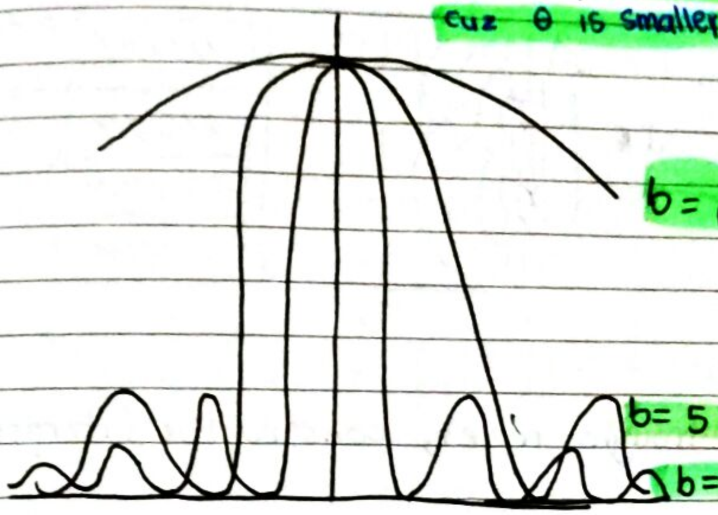


5) **Changing b**

The shape of the diffraction pattern depends on the ratio of the slit width b to the wavelength λ. $\frac{\lambda}{b} = \theta$ or ~~the distance between central max and first minimum~~

The bigger the b, the closer the maxes and mins.

cuz θ is smaller in $\frac{\lambda}{b\uparrow}$

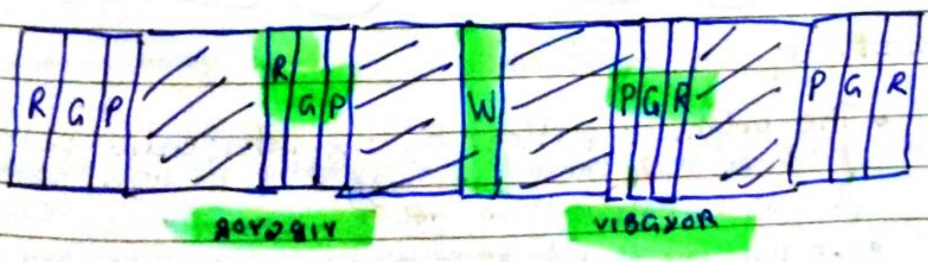


The smaller the λ is, the closer the maximas & minimas also.

So, when θ is smaller, the maximas and minimas are closer.

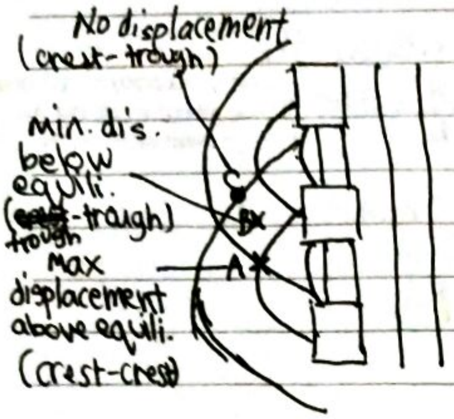
Purple light, $b = 10\lambda$
Green light, $b = 5\lambda$
Red light, $b = \lambda$

So, when white light is projected through a single slit, central maxima is white, then in ~~first~~ other maximas - purple is first, closest to center, green second and red third



9.3 Interference

1) Double slit interference

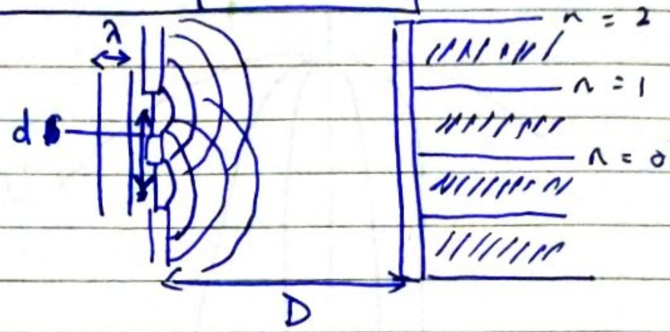


This was discovered by Thomas Young in 1801.

$s = \frac{\lambda D}{d}$	Young's double slit experiment

s = distance between maximas.

d = slit spacing



When 2 crests or troughs meet, constructive interference.

When a crest and trough meets, destructive interference.

Example. Coherent light with wavelength 675nm is incident on opaque card having 2 vertical slits separated by 1.25mm. Screen 4.50 m away from card. What is the distance between central and first maximum?

$\lambda = 675 \times 10^{-9}$ m

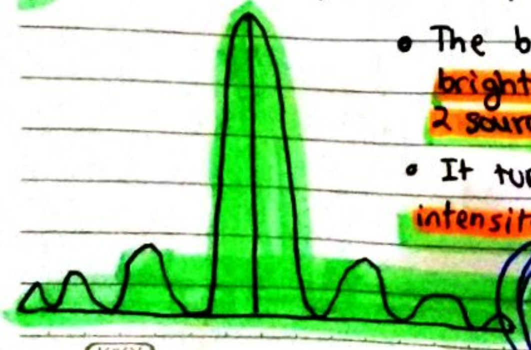
$d = 0.00125$ m

$D = 4.50$ m

$$s = \frac{\lambda D}{d} = \frac{675 \times 10^{-9} \times 4.5}{0.00125}$$

$s =$ ~~2.43 m~~ 0.00243 m

2) Double-slit ~~inpeference~~ interference – intensity



- The brightest portions of the interference pattern are brighter than those in single slit because there are 2 sources, rather than one.
- It turns out that if N is the number of slits, and I_1 is the intensity of the single-slit central maximum, then,

$I_N = N^2 I_1$	Intensity of central maximum for N slits

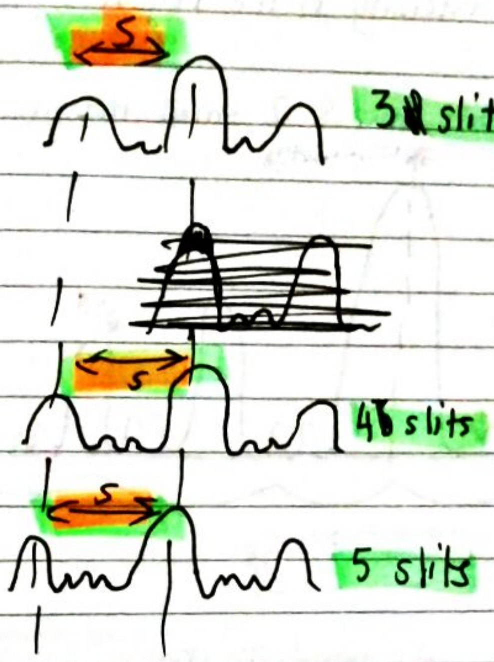
Not in Data Booklet

Date

So for 2 slits, intensity of central maxima would ~~be~~ be $4I_1$.

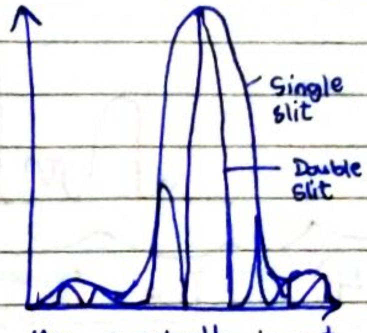
3) Multiple-slit interference

The separation $s = \frac{\lambda D}{d}$ between maximas remains even when the number of slits multiply.



d is always same. Distance

So s also.



~~So the s actually is not.~~

We take the first minima or max as the single slit pattern one. Not the smaller & narrow fringes.

The intensity increases as slits increase. More light gets through. Narrower/Sharper maximas.

Example

Light with $\lambda = 675 \times 10^{-9}$ m is projected onto four vertical slits equally spaced at 16.875 μm between slits. Slit widths are 6.75 μm. The resulting diffraction pattern is projected on a wall that is 5.00m away from slits.

a) Find the separation between bright points in the pattern.

~~$s = \frac{(16.875 \times 10^{-6} + 6.75 \times 10^{-6})(675 \times 10^{-9})(5)}{}$~~

~~$= \frac{(675 \times 10^{-9})(5)}{0.000023625} = \frac{0.2m}{0.141m}$~~

~~16.875×10^{-6}~~

Solution

$S = \frac{\lambda D}{d} = \frac{675 \times 10^{-9} \times 5}{16.875 \times 10^{-6}}$

$s = 0.2m = 0.200m$

b) Determine the width of the brightest central region of the pattern.

$\theta = \frac{2\lambda}{b} = \frac{d}{D}$

$d = \frac{2\lambda D}{b}$ $s_{min} = \frac{\lambda D}{b} \times 2$

$d = \frac{675 \times 10^{-9} \times 2 \times 5}{6.75 \times 10^{-6}}$

$d = 1m = 1.00m$

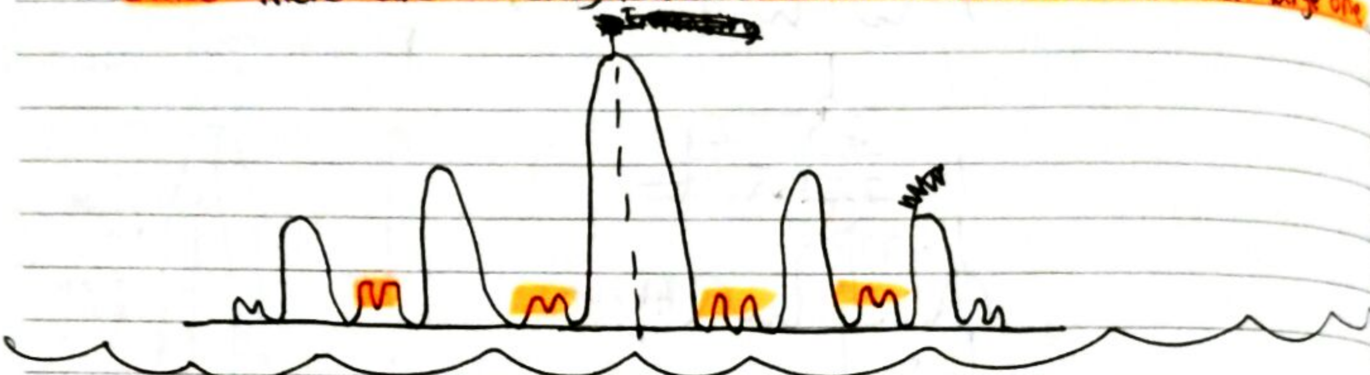
KIKY

c) Approximately how many bright points will fit in this central region?

$$\frac{1}{0.2} = 5 \text{ points}$$

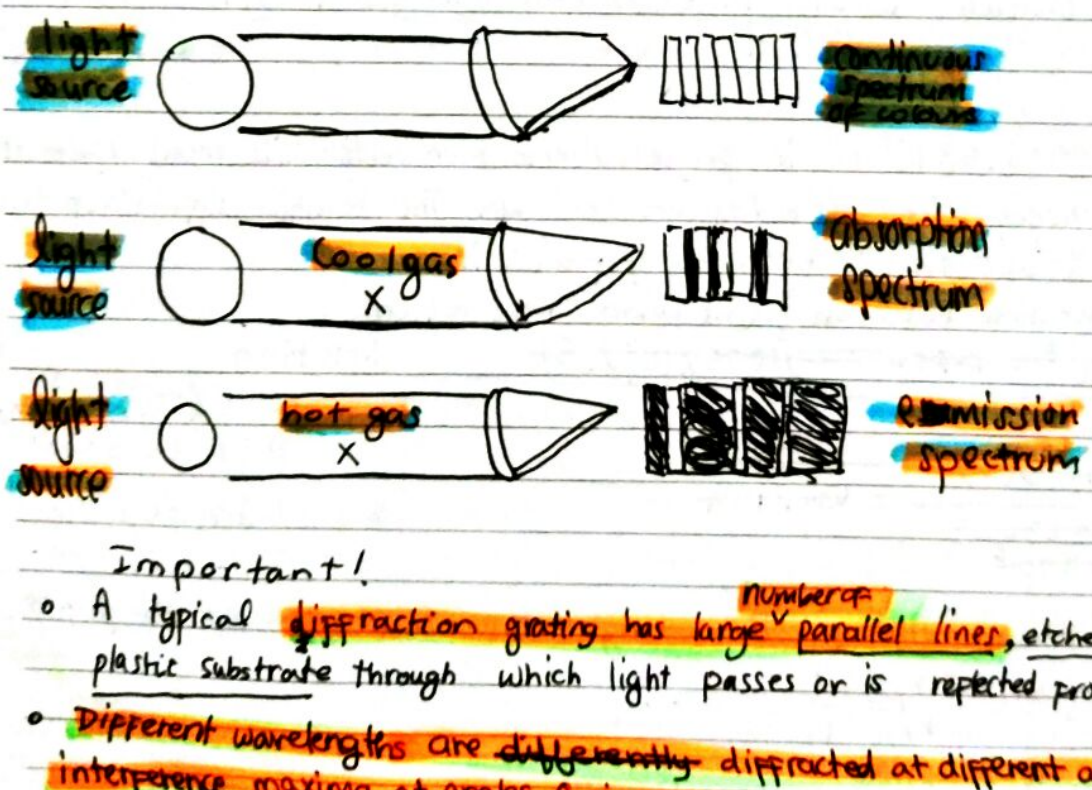
d) Sketch the pattern's intensity in the region of the central maximum.

Since there are 4 slits, there's 2 small maximas between each large one



4) The diffraction grating

- Diffraction gratings are used to make optical spectra.



Important!

- A typical diffraction grating has large number of parallel lines, etched in a glass or plastic substrate through which light passes or is reflected from.
- Different wavelengths are ~~differently~~ diffracted at different angles producing interference maxima at angles θ given by

$n\lambda = d\sin\theta$	diffraction grating maxima locations

n = order of maxima. n = 0 is central max. n = 1 is on either side of central max.

d is the distance between lines.

θ = angle of maximum

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Example

A diff. grating that has 750 lines per mm is illuminated by a monochromatic light, normal to the grating. Third order max. observed at 56° to the straight-through direction. Determine wavelength of the light.

$n\lambda = d\sin\theta$

$3\lambda = d\sin\theta$

750000 lines per m

Distance between lines $= \frac{1}{750000}$

$3\lambda = \frac{\sin 56°}{750000}$

$\lambda = \frac{\sin 56°}{2250000}$

$\lambda = 3.7 \times 10^{-7}$ m

$n\lambda = d\sin\theta$

$3\lambda = \frac{1}{750000}\sin 56$

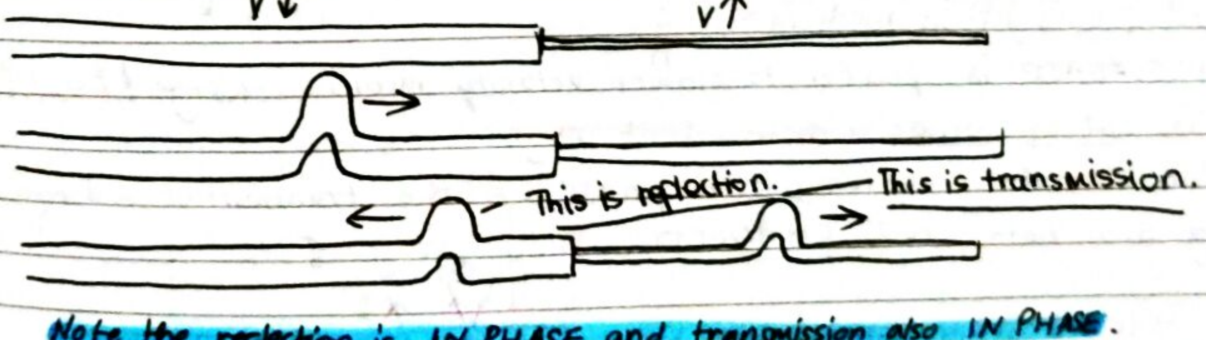
$\lambda = \frac{\sin 56}{2250000}$

$\lambda = 3.7 \times 10^{-7}$ m

5) Thin film interference

First, understand how waves are reflected at the boundary of a heavy and light rope.

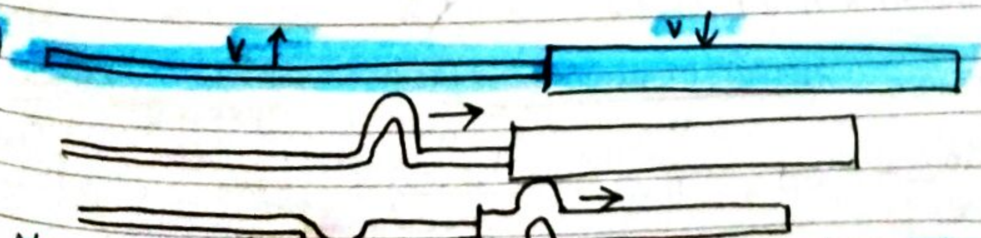
1)



Note the reflection is IN PHASE and transmission also IN PHASE.

Note. Velocity v is less in heavy rope than in light rope.

2)



Note Reflection is OUT OF PHASE and transmission still in PHASE.

KIKY

- Three properties to learn from this:

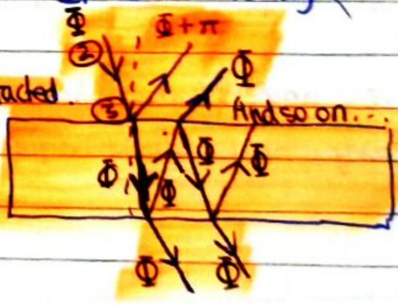
1) If a wave enters a boundary from a ~~faster~~ slower wave speed medium, it will reflect IN phase. Just like case 1).

2) If a wave enters a boundary from a faster wave speed medium it reflects OUT OF PHASE. Just like case 2).

3) In any case, transmitted wave through the boundary is always IN PHASE with the original pulse.

With these 3 properties, you can solve all thin film interference problems.

1) Consider a thin film of glass of thickness d with refractive index n.
2) Light of wavelength λ is normally incident on film. It is shown at a slight angle to show the reflection and refraction. It is actually at 90° to the film.
3) At top surface, some light is reflected & some refracted.
4) This happens at each subsequent surface.

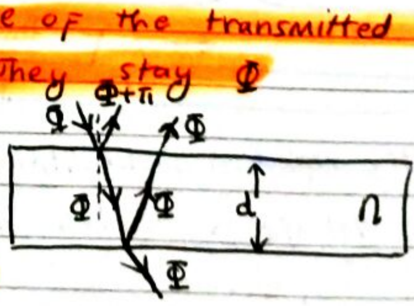


5) Speed of light in air $= c$

Speed of light in medium $= \frac{c}{n}$

6) TOP SURFACE is faster to slower velocity medium change! So, reflected ray is out of phase with incident ray.
7) Once inside medium, the phase of the transmitted and reflected ray are both NOT CHANGED. They stay Φ
8) In this diagram, light travels distance $2d$ INSIDE the film.

REMEMBER! Angle of incidence is actually 0.



9) Speed in medium $= \frac{\text{speed in vacuum}}{n} = \frac{c}{n}$ speed $= \frac{\text{distance}}{\text{time}}$

$$\frac{\text{distance}}{\text{time}} = \text{speed}, \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{2d}{\frac{c}{n}} = \frac{2dn}{c}$$

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So, we see that the ~~speed~~ time inside a medium for a typical crest to make it through the medium is

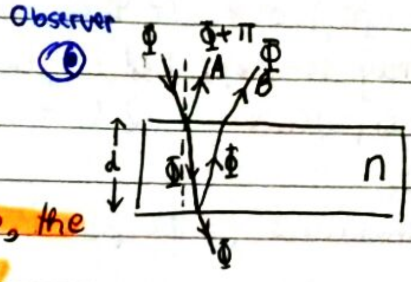
$$t = \frac{2dn}{c}$$

t = time for a crest to make it through the medium.
d = width of medium
n = refractive index of medium
c = $3 \times 10^8 \, ms^{-1}$

10) Outside the medium, it travels m wavelengths λ in

$$\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{m\lambda}{c}$$

(distance = $m\lambda$)



11) Now read carefully!

The angle is actually 0. So, the waves "go into each other".
A & B are out of phase. So, when they rejoin at the top surface, they destructively interfere if their transit times are equal:
B partially cancels A.

$$\text{time} = \frac{2dn}{c} = \frac{m\lambda}{c}$$

$2dn = m\lambda$	thin film destructive interference

m = no. of wavelengths travelled outside medium. Order.
d = width of medium
n = refractive index

12) If the following relation holds, the ~~rejoing~~ rejoining rays interfere constructively:

$$\text{time} = \frac{2dn}{c} = \frac{(m+\frac{1}{2})\lambda}{c}$$

$2dn = (m+\frac{1}{2})\lambda$	thin film constructive interference

- We assume observer on the same side as light source.
- We also assume only one ray was reflected π out of phase.

13) If both rays are π out of phase or observer is on the "away side", reverse the formulas. In latter case, observer sees the rays that come out in phase from the bottom side of the film.
$2dn = m\lambda$, constructive — B enhances A.
$2dn = (m+\frac{1}{2})\lambda$, destructive

Practice

A film of oil with refractive index 1·40 floats on a puddle of rain water with n = 1·33. Puddle is illuminated by sunlight. When viewed from near normal incidence, a particular region of oil film has orange colour, having wavelength 575 nm.

a) Explain how refractive indices of the air, oil & water play a part in producing the orange colour.

When light is incident at the oil, since $n_{oil} > n_{air}$, the reflected ray is π out of phase. At the oil-water boundary, since $n_{oil} > n_{water}$, ray is reflected in phase. As we can see orange, there is constructive interference $\therefore$ $2dn = (m + \frac{1}{2})\lambda$

b) Calculate the possible thicknesses of the film in the orange region.

thickness = d

$2dn = (m + \frac{1}{2})\lambda$

$2d(1{\cdot}40) = (m + \frac{1}{2})(575 \times 10^{-9})$

$d = \dfrac{(m+\frac{1}{2})(575 \times 10^{-9})}{2{\cdot}80}$

~~$2dn = m\lambda$~~

$2dn = (m + \frac{1}{2})\lambda$

$M = 0, 1, 2, \ldots$

Possible d

$= 1{\cdot}03 \times 10^{-7}\ M,\ 3{\cdot}08 \times 10^{-7}\ M,\ 5{\cdot}13 \times 10^{-7}\ M \ldots$

c) Calculate minimum thickness of film in orange region.

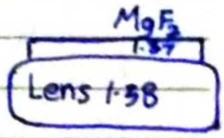
$d = \dfrac{(0 + \frac{1}{2})(575 \times 10^{-9})}{2{\cdot}80}$

$d = 1{\cdot}03 \times 10^{-7}\ M$

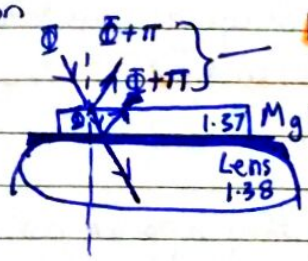
Date

Example

Magnesium Fluoride MgF_2 has $n = 1.37$. It is applied ~~on~~ in a thin layer ~~of~~ ~~opti~~ over an optical lens made of glass with $n = 1.38$. What thickness should it be so that light of wavelength 528 nm is not reflected from the lens?



Solution



Both reflections are in PHASE $\Phi + \pi$. This makes them constructive.

To have destructive interference, we need the thickness of MgF_2 layer be out of phase & satisfy = $2dn = \left(m + \frac{1}{2}\right)\lambda$

$$2(d)(1.37) = \frac{1}{2} \times 528 \times 10^{-9}$$

$$d = \frac{528 \times 10^{-9}}{4 \times 1.37}$$

$$d = 9.64 \times 10^{-8} \text{ M}$$

$$2dn = \left(m + \frac{1}{2}\right)\lambda$$

$$2d(1.37) = \left(m + \frac{1}{2}\right)(528 \times 10^{-9})$$

$$1.37d = \frac{528 \times 10^{-9}}{4}$$

$$d = 9.64 \times 10^{-8} \text{ M}$$

If you read "film", the question is ^Most likely^ about thin-film interference.

$\frac{d}{dx}\left[|x-2|+1\right]$ $x<2$

0

$\frac{-x+2}{-1}$

9.4 - Resolution

Data booklet:

$$\theta = \frac{1.22\lambda}{b}$$

$$R = \frac{\lambda}{\Delta\lambda} = mN$$

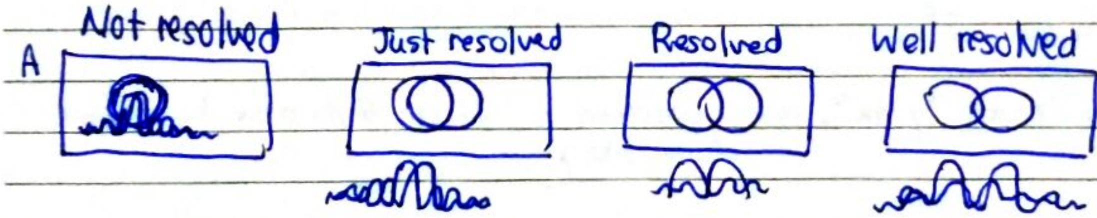
Resolution = the ability of an ~~mass~~ imaging system to be able to produce 2 separate distinguishable images of two different objects.

When you are far from 2 objects, they appear as one.

1) The resolution of monochromatic two-source systems

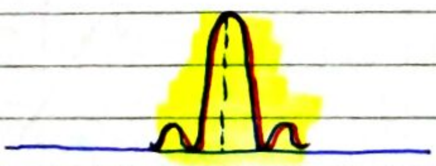
Headlights of a truck can be an example.

FAR → CLOSER

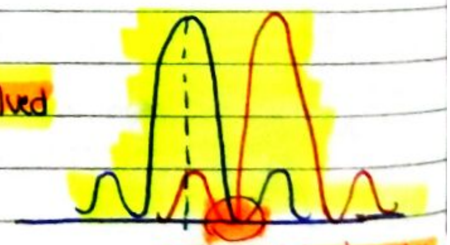


See the intensities more closely:

1) Not resolved

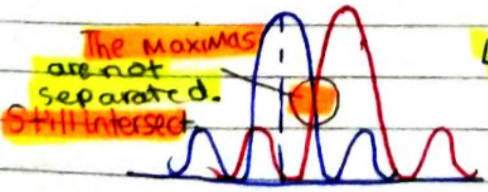


3) Resolved

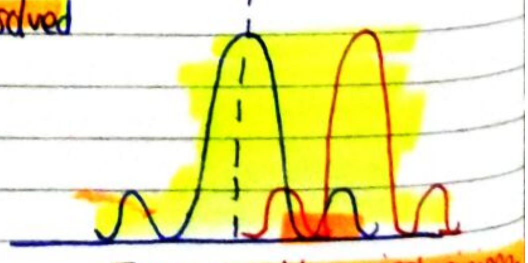


Intersect just after central maxima. At first minima.

2) Just resolved



4) Well resolved



Intersect after first minima.

Date

Two sources of light are just resolved ~~using an~~ using an aperture if the first minimum of diffraction pattern of one of the sources falls on the central maximum of the diffraction pattern of the other source. See number 2.

This criterion yields these formulas for minimum angular separation: 

$\theta = \frac{\lambda}{b}$ (θ in rad)	b □ b	Rayleigh criterion for SQUARE apertures
$\theta = \frac{1.22\lambda}{b}$ (θ in rad)	○ b	Rayleigh criterion for CIRCULAR apertures

viewer, not object's shape. Like an eye. Telescope.

Practice

A car's headlights ($\lambda = 530nm$) are 0.75 m apart. At what max distance can they just be resolved by the human eye. (diameter = 2.5 mm)?

Human pupil is circular.

$\theta = \frac{1.22\lambda}{b}$

$\theta = \frac{1.22(530 \times 10^{-9})}{0.0025}$

$\theta = 2.59 \times 10^{-4}$ rad

$\theta = \frac{0.75}{\text{distance}}$

$2.59 \times 10^{-4} = \frac{0.75}{d}$

$d = 2900m$

$= 2.9km$

$\theta = \frac{1.22\lambda}{b}$

$\theta = \frac{1.22(530 \times 10^{-9})}{0.0025} = 2.59 \times 10^{-4}$

$\theta = \frac{1.22\lambda}{b} = \frac{d}{D}$

d = distance between objects

D = distance between aperture & objects

θ = minimum angular separation needed to resolve the objects

$2.59 \times 10^{-4} = \frac{0.75}{d}$

$d = 2900m \approx 2.9km$

Practice

A 68-meter diameter radio telescope is receiving radio signals of $\boxed{2.0\ GHz}$ from 2 stars 75 lightyears (ly) away and separated by 0.045 ly. Frequency use $\frac{c}{f} = \lambda$

a) Can the telescope resolve the images of the two stars?

$\theta = 1.22\frac{\lambda}{b} = \frac{\frac{c}{f}}{b} = \frac{\frac{3 \times 10^8}{2 \times 10^9}}{68} = \frac{0.15}{68} = \theta = 2.2 \times 10^{-3}$ rad

So, the θ needed is: $\frac{1.22\lambda}{b} = \frac{1.22\frac{c}{f}}{b} = \frac{1.22(0.15)}{68} = 2.7 \times 10^{-3}$ rad

For stars ~~Actual~~ θ: $\frac{0.045}{75} = 6 \times 10^{-4}$ rad.

No need to convert light years as the units cancel! Don't be premature.

∴ No. The angular separation of the stars is too small for the telescope to resolve.

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Date

b) If another identical radio telescope is located 350 m away and it can be used in concert with the first one to create a single telescope having an effective diameter of 350 m, can the pair resolve the two stars?

For telescope
~~Required~~ $\theta = \frac{1.22\lambda}{b} = \frac{(0.15)1.22}{350} = \cancel{(1.43 \times 10^{-3}) \text{rad} (1.22) = 1.74 \times 10^{-3} \text{rad}}$
$= \underline{0.000523 \text{ rad}}$

For stars
~~Actual~~ $\theta = \frac{0.045}{75} = \underline{6 \times 10^{-4} \text{ rad}}$

Yes ~~No~~, the pair ~~still cannot~~ can resolve the 2 stars because the angular separation between the stars ~~is still too small larger than the~~ large enough for the telescope to resolve.

FYI - important!

So, remember that θ for stars or the 2 ~~objects~~ sources ~~should~~ must be higher than the θ from ~~telescope or eyes~~ optical device for the sources to be resolved.

$$\theta_{source} = \frac{\text{distance between each other}}{\text{distance from } \cancel{\text{viewer eyes}} \text{ optical device}}$$

θ is minimum

$$\theta_{\cancel{eyes}\ opt} = \frac{\lambda}{b} \text{ or } \frac{1.22\lambda}{b}$$

$\theta_{\cancel{eyes}\ opt.} \leq \theta$ source, if resolved

What are the 2 ways to increase resolution of an optical device.

Well, we have to decrease θ in $\boxed{\theta = \frac{\lambda}{b}}$. So,

1) ~~Increase~~ Decrease wavelength ~~of~~ received from sources
2) ~~Decrease~~ Increase diameter ~~of~~ optical device

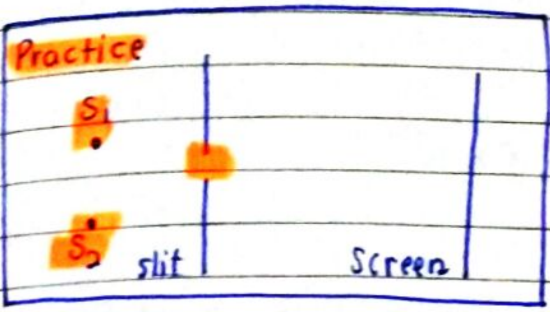
(1) Decrease wavelength received
(2) Increase diameter of optical device

Obviously a 200 inch diameter telescope has a better resolution than a 2 inch diameter telescope.

~~2 slit~~ 2 source 1 slit Date

~~PRACTICE~~

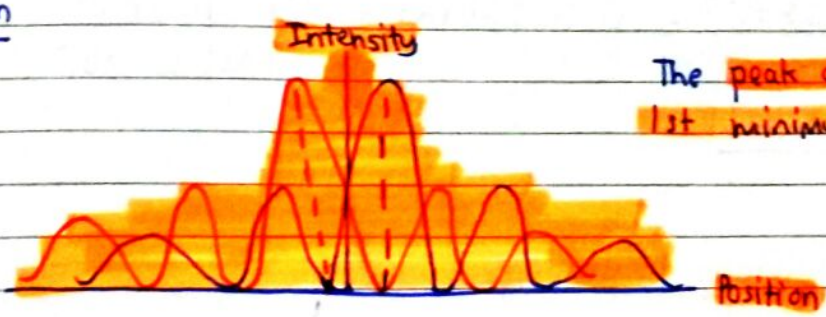
~~An electron microscope (EM) uses electrons which have been accelerated under a p.d. of 750V. Explain why the EM has better resolution than the light microscope.~~



Q a) Light from 2 monochromatic distant point sources S_1 and S_2 is incident on a narrow slit. After passing through, the light is incident on a ~~slit~~ screen.

Draw the intensity distribution on screen when images S_1 and S_2 are just resolved according to Rayleigh criterion.

Solution



The peak of one matches the 1st minimum of the other.

A woman watches an approaching car at night. Aperture of eyes 3.0mm. Headlamps separated by 1.2m and emit light of wavelength 400nm.
Calculate distance of car from woman at which images of the 2 headlamps are just resolved.

$$\frac{1.22\lambda}{b} = \frac{1.2}{D}$$

$$\frac{1.22(400\times10^{-9})}{0.003} = \frac{1.2}{D}$$

$$\theta = \frac{1.22\lambda}{b} = \frac{(400\times10^{-9})1.22}{0.003} = 1.6\times10^{-4}\text{ rad}$$

$x = 7.4$ km

$$\frac{1.2}{x} = 1.6\times10^{-4}$$

$$x = \frac{1.2}{1.6\times10^{-4}}$$

$x = 7.4$ km

Q) Pluto is 4.5×10^{12} m from Earth and Pluto's diameter is 2.3×10^{6} m. The average wavelength of light received by Earth from Pluto is 500 nm.
Deduce, whether ~~human eye~~ sees Pluto as a disc or a point source of light.

$$\frac{1.22(500\times10^{-9})}{b} = \frac{2.3\times10^{6}}{4.5\times10^{12}}$$

$b = 1.2$ m

Too large diameter for an eye. Only as point-source.

$$\theta = \frac{2.3\times10^{6}}{4.5\times10^{12}} = 5.1\times10^{-7}\text{ rad}$$

$$\theta = \frac{\lambda}{b}, \quad b = \frac{(500\times10^{-9})1.22}{5.1\times10^{-7}} = \cancel{0.98m}\ 1.2\text{m}$$

∴ The diameter is too large for an eye. Hence, pluto is seen as a point source.

KIKY

Date

Practice

Two binary stars emit radio waves of wavelength 6.0×10^{-2}. Waves received by a radio telescope of diameter 120m. The two stars are just resolved if their minimum angular separation in radians is of the order:

A. 2×10^{4} B. 2×10^{2} C. 5×10^{-2} D. 5×10^{-4}

$\frac{1.22(0.06)}{120} = \frac{0.06}{100} = 0.0006$
6×10^{-4}

Solution

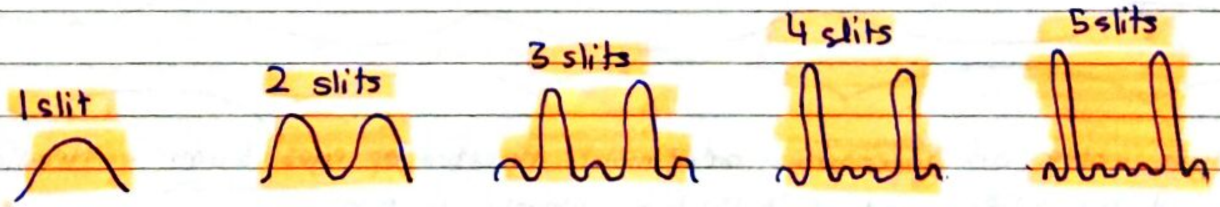
$\theta = \frac{1.22\lambda}{b}$ ($\frac{\lambda}{b}$ also works because they are only asking for the order)

$\theta = \frac{1.22 \times (6 \times 10^{-2})}{120} = 0.00061 \text{ rad} = 6.1 \times 10^{-4} \text{ rad}$

Order is 10^{-4}.

$\boxed{\text{D. } 5 \times 10^{-4}}$

2) Resolvance of diffraction gratings



Observe the intensity patterns for multiple-slit interference

1. The more slits, the higher the intensity of primary maxima.
2. The more slits, the narrower the primary maxima.

~~widths of~~ There are hundreds of slits, so the widths of the primary maxima are very small indeed.

Resolution for a grating is proportional to number of ~~slits~~ lines N.

Formula for Resolution in diffraction grating

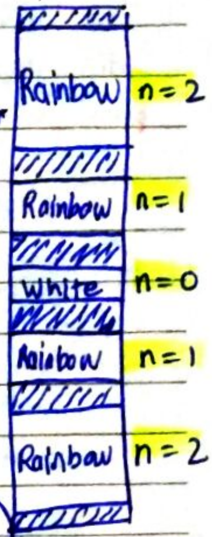
The resolvance R of a diffraction grating is defined as the ratio of the average λ of two wavelengths to their difference $\Delta\lambda$. Thus

$\boxed{R = \frac{\lambda}{\Delta\lambda}}$ Resolvance of a diffraction grating

Resolvance = the measure of how well a diffraction grating can separate two wavelengths.

No unit for R.

- Grating resolution is proportional to N, the total lines illuminated by the incident beam of light.
- Observe the diffraction pattern on the right. We see that a natural consequence of diffraction is the spreading out of the pattern for higher orders. So, the natural separation of wavelengths increases with order.



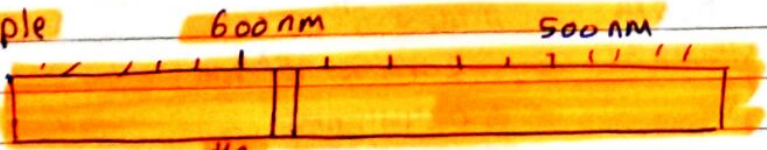
n is m. No difference. Both mean order of diffraction.

Without proof, resolvance R for each order of diffraction m is given by $R = Nm$, where N is total number of slits illuminated by the beam.

$\boxed{R = Nm}$ Resolvance, Number of slits illuminated, order of diffraction

$R = \dfrac{\lambda}{\Delta\lambda} = Nm$	Resolvance of a diffraction grating

Example



Two lines of the sodium emission spectrum are visible, with wavelengths 589.0 nm & 589.6 nm. A diffraction grating is illuminated with a beam of this light having a width 0.2500 mm.

a) Resolvance of this grating?

$\dfrac{589 + 589.6}{2} = 589.3 \text{ nm}$

$R = \dfrac{\lambda}{\Delta\lambda} = \dfrac{589.3}{0.6} = 982.2$

$\dfrac{589.3}{0.6} = 982.2$

b) Find the minimum no. of lines under the beam needed for the resolvance of the order 2 spectrum.

$982.2 = 2N$

$N = 491.1$

$982.2 = Nm$

$m = 2$

$\dfrac{982.2}{2} = 491.1$

c) Find minimum lines needed per mm in this particular diffraction grating.

Lines per mm $= \dfrac{491.1}{0.25} = 1964$ lines

$\dfrac{491.1}{0.25} = 1964$

Date

Practice

A beam of light has avg. λ 550.0 nm, It is incident on 1.50 cm wide dif. grating have 500 lines per mm. Calculate smallest difference in wavelength that can be resolved in the 3rd order and find resolvance of the grating.

~~$\lambda = 550$ nm~~

~~$R = \frac{\lambda}{\Delta\lambda} = Nm$~~

~~$\Delta\lambda = \frac{550}{3 \times 0.0005}$~~

~~$\Delta\lambda =$~~

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

$$\cancel{\frac{550.0 \times 10^{-9}}{\Delta\lambda}} = \cancel{500000 \times 3}$$

$$\cancel{\Delta\lambda =}$$

$$\text{lines/mm} = \frac{N}{\text{width}}$$

$$500 = \frac{N}{15}$$

$$7500 = N$$

$$\Delta\lambda = \frac{\lambda}{Nm} = \frac{550 \times 10^{-9}}{7500 \times 3} = 2.44 \times 10^{-11}\ m$$

$$= 2.44 \times 10^{-2}\ nm$$

$$= 0.02444\ nm$$

$$R = \frac{550.0}{0.02444} = 2.25 \times 10^{3} = 2.250 \times 10^{4} = 22500$$

OR

$$R = Nm = 7500 \times 3 = 22500$$

Just more information in case!

- CDs cannot hold more info. than DVDs because a DVD player must have a laser of higher frequency (smaller λ) than a CD player. With higher f. smaller pits can be resolved in DVDs.

	CD	DVD	Blu-ray disc
	$\lambda = 780$nm	$\lambda = 650$nm	$\lambda = 405$nm
Storage	700 MB	4.7GB	25GB

Take care of the difference between λ and f.

9.5 Doppler effect

- Doppler effect describes the wavelength/frequency shift during relative motion.

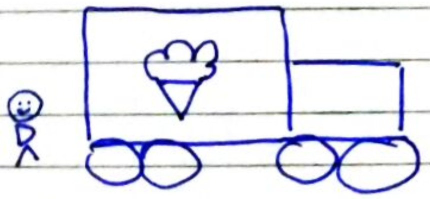
Application: Expansion of universe, weather reports, medicine

- Situations include radars and medical physics.
- Significance for the the red-shift in the light spectra of receding galaxies

1) The Doppler effect - moving source

- The Doppler effect explains frequency change caused by moving sources or moving observers.

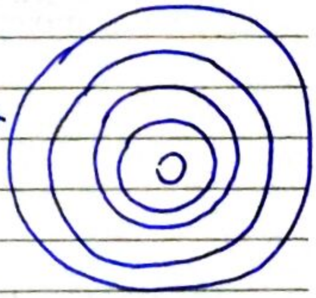
- When a sound source of frequency f approach you at speed u_s, its wavefronts bunch together. You hear a frequency f' which is higher. $(f' > f)$
- When sound source will recede from you, the wavefronts stretch out and you hear frequency f' which is lower. $(f' < f)$



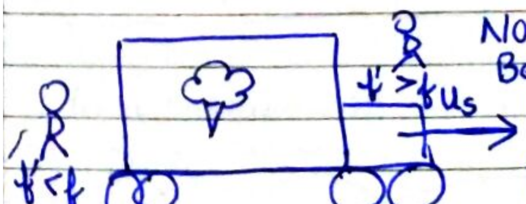
The truck is stationary at the moment

$u_s = 0$

The cross-section of wave fronts is symmetric.



Dobson hears same frequency wherever he stands.



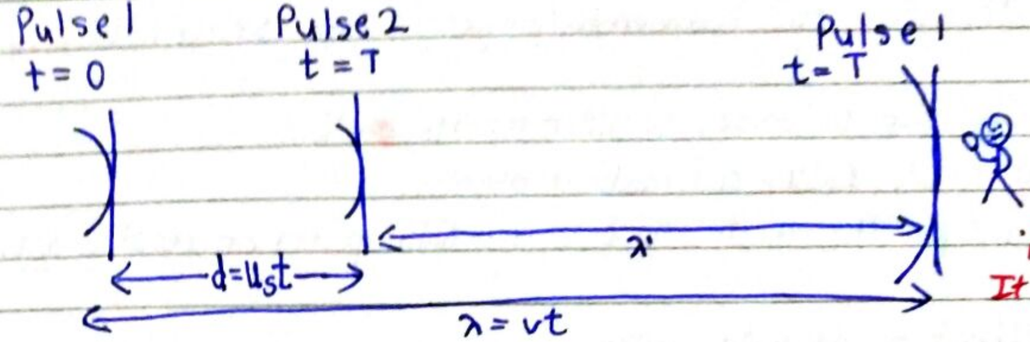
Now truck moves at u_s.
Bell rings at same rate as before.
But, wavefronts bunch up in front and separate at the back.

- Reason: the truck moves forward during each successive spherical wave emission.
~~(u_s is actually~~ (the speed of wavelengths is speed of sound, bunched or not!)

Dobson hears different frequency now.
If front of truck, $f' > f$.
If back of truck, $f' < f$.

Date

Let's look at two successive pulses for a source moving at u_s:



Remember how this diagram works. There can be questions about what the observed λ is.
It is $\lambda' = \lambda - u_s t$ or
$\lambda' = \lambda + u_s t$ (away)

- Pulse 1 reaches Dobson in time T. In that time next pulse is emitted by truck at d distance in front. Pulse 1 has travelled λ in time t = T.
- λ is the wavelength of sound. ~~It is velocity x Period (T)~~
- But observer detects wavelength λ'. ($\lambda' < \lambda$) (which means he detects higher f)
- $d = u_s T$. d is distance between emission of the pulses.

$\cancel{\lambda = d + \lambda'}$ $\quad \lambda = d + \lambda'$

$\cancel{\lambda' = d - \lambda}$ $\quad \lambda' = \lambda - d$

$$\lambda' = T(v - u_s)$$

$$f' = \frac{v}{\lambda'} = \frac{v}{T(v-u_s)}$$

$$f' = \frac{1}{T} \times \frac{v}{v-u_s}$$

$f' = f\left(\frac{v}{v \pm u_s}\right)$	approach ($-u_s$) recede ($+u_s$)	Doppler effect in moving source

If source recedes, u_s should be added.
Use logic! When f' should be higher, make denominator smaller and vice-versa.

Example Car horn has f 520 Hz. Car travels rightward at 25 ms^{-1} and speed of sound is 340 ms^{-1}.

a) frequency heard by driver?

$$f' = f\left(\frac{v}{v \pm u_s}\right) = 520\left(\frac{340}{340}\right)$$

Oh wait!

Driver is in the car. Relative motion is 0.

Driver hears 520 Hz.

Date

b) Frequency heard by observer on left side?

$$f' = f\left(\frac{v}{v+v_s}\right) = 520\left(\frac{340}{365}\right) = 480\text{ Hz}$$

c) Frequency heard by observer on right side of the car?

$$f' = f\left(\frac{340}{315}\right) = \frac{520\times 340}{315} = 560\text{ Hz}$$

2) The Doppler Effect – Moving observer

When both the source & observer are stationary, $f' = f$.

But if observer is moving TOWARD the source at v_0, Frequency observed is higher. $(f' > f)$

If observer moves AWAY, frequency observed is lower. $f' < f$.

$f' = f\left(\frac{v \pm v_0}{v}\right)$	v is speed of sound.	Doppler Effect
approach$(+v_0)$, recede$(-v_0)$		Moving observer

Always use logic when picking the + – sign.

Example

Car horn frequency is 520 Hz. Speed of sound is 340 ms^{-1}.

a) An observer approaches car at 25 ms^{-1}. What frequency is heard by him?

$$f' = f\left(\frac{v \pm v_0}{v}\right) = 520\left(\frac{340+25}{340}\right) = 560\text{ Hz}.$$

$$520\left(\frac{340+25}{340}\right) = 560\text{ Hz}$$

b) Recedes at 25 ms^{-1}. Frequency?

$$f' = 520\left(\frac{315}{340}\right) = 480\text{ Hz}.$$

$$520\left(\frac{315}{340}\right) = 480\text{ Hz}$$

KIKY

3) The Doppler effect - light (optional derivation)

Here, $v = c.$ $3.0 \times 10^{8}\ ms^{-1}$

Thus, $f' = f\left(\frac{v}{v \pm u_s}\right)$ becomes

$$f' = f\left(\frac{c}{c \pm u_s}\right)$$
$$= f\left(1 \pm \frac{c}{u_s}\right) = f\left(1 \pm \frac{u_s}{c}\right)^{-1}$$

And, $f' = f\left(\frac{v \pm u_0}{v}\right)$ becomes

$$f' = f\left(\frac{c \pm u_0}{c}\right)$$
$$= f\left(1 \pm \frac{u_0}{c}\right)^{1}$$

In binomial theorem, there is $(1+x)^n$

Take $x = \frac{u_0}{c}$ or $\frac{u_s}{c}$

$|x|$ is always less than 1. Both formulas reduce to:

So, $f' = f\left(1 \pm \frac{u}{c}\right) = f \pm \frac{fu}{c} \rightarrow \frac{\Delta f}{f} = \frac{\pm u}{c}$

$$f' - f = \Delta f$$

~~That is why~~ So, $\frac{\Delta f}{f} = \frac{\pm u}{c}$

u is either speed of source or observer, or the relative speed between both

v is same as u. IBO just makes it v.

$$\frac{\Delta f}{f} = (f' - f)/f = \left(\frac{c}{\lambda'} - \frac{c}{\lambda}\right)/\left(\frac{c}{\lambda}\right) = \left(\frac{\lambda}{\lambda'} - 1\right) = \frac{\lambda - \lambda'}{\lambda'}$$
$$= \frac{\Delta\lambda}{\lambda}$$

$\frac{\Delta f}{f} = \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$	Doppler effect for light	v is relative speed between source & observer.

The derivation is just for understanding.

Date

Example

A star moves away from us at $5.6 \times 10^6 \, ms^{-1}$. It has a known absorption spectrum line that should be located 520 nm on an identical stationary star. Where is this line located on the moving star?

$$\frac{\Delta\lambda}{520 \times 10^{-9}} = \frac{5.6 \times 10^6}{3 \times 10^8}$$

$\Delta\lambda = 9.7$ nM

Since it moves away.

$\lambda \uparrow$, $f \downarrow$

$520 + 9.7 = 529.7$ nM

≈ 530 nM

Don't be scared. Get the important part.

It is about wavelength. $\lambda = 520$nm for stationary star.

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = \frac{5.6 \times 10^6}{30 \times 10^8} = 0.0187$$

$\boxed{\lambda \text{ increases when go away, and decreases when come close.}}$

$$\frac{\Delta\lambda}{\lambda} = 0.0187$$

$$\frac{\Delta\lambda}{520} = 0.0187$$

$\Delta\lambda = 9.7$ nm

We have to find λ'. Since it moves away from us, λ is higher.

$$520 + 9.7 = 529.7\text{nm} = 530\text{nm}.$$

So, Doppler effect for light usually has questions about Space.
"red-shift" is shown as evidence of expanding universe.

4) Applications of Doppler effect

Radar gun (used by police). Measures difference in f between the emitted radar beam, and the reflected and returned beam. It finds velocity of cars.

Doppler Ultrasound test: uses reflected sound waves to see how blood flows through a blood vessel. Detects blockages, major arteries, blood flow, clots etc.

5) Solving problems

Q) A source emits soundwaves of λ, period T and speed v, at rest. It moves away from observer at speed V. The wavelength of the sound waves, as measured by observer is?

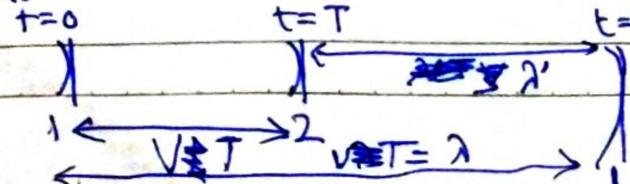
But, here sources goes away. So, λ' is $\boxed{VT + \lambda}$

$\lambda = vT$

$\lambda' = vT + VT$

$\boxed{\lambda' = \lambda + VT}$

Remember the diagram before?



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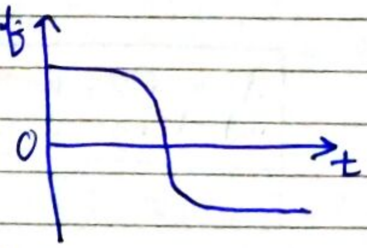
Q) A source S, at constant speed, emits sound of constant frequency. The source passes a stationary observer O as shown:

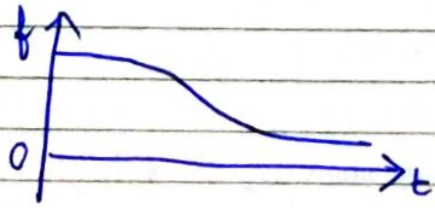
What shows the variation of frequency f observed by O as source S approaches and passes O?

A.

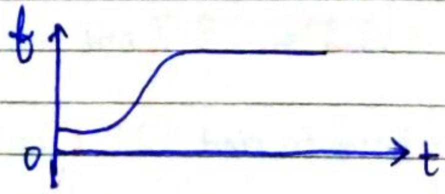


B.

C.

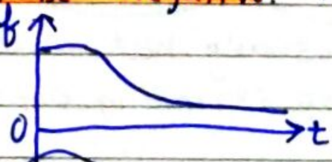


D.



Solution.

As the sources goes, f decreases. B and D are wrong.
Frequency can't be negative. A is wrong.
C is correct.



Q Why does an observer hear higher frequency than actual frequency when a sound source approaches? Source is truck.

A. Wavefronts received by observer are closer together than wavefronts received by driver (source).

B. Speed of wavefronts is greater for observer. v is constant (340ms⁻¹)

C. Speed of wavefronts is lesser for observer. v is constant

D. wavefronts recieved are further apart than those received by driver. Nope

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Date

Doppler effect is:

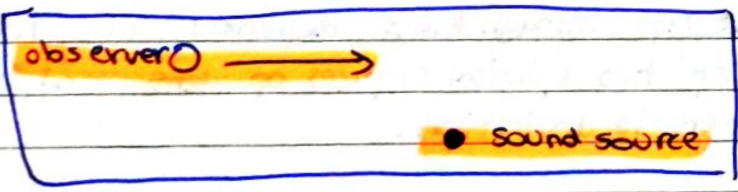
A. Change in frequency of light due to motion of source. ~~light~~
Frequency is unchanged. It is observed frequency. It is relative motion.

B. Change in frequency of light due to relative motion of S and O.

C. Change in observed frequency of light due to relative motion of S and O.

D. Change in observed frequency of light due to change in velocity of source light.
V is unchanged.
Nothing to do with Doppler Effect.

Q) Sound of f_0 is emitted by source. Observer O travels towards it at $0.1v$, where v is speed of sound.



Which of these given relationship between f_0 and the frequency f observed?

A. $f = 1.1 f_0$ B. $f_0 < f < 1.1 f_0$ C. $f = 0.9 f_0$ D. $f_0 > f > 0.9 f_0$

$\cancel{f' = f\left(\frac{v+v_0}{v}\right)}$

$\cancel{f = f_0\left(\frac{v+0.1v}{v}\right)}$

$\cancel{f = 1.1v\ 1.1 f_0}$

$f = f_0\left(\frac{v+0.1v}{v}\right)$

$f = 1.1 f_0$, if O and S are in straight line.

But, they are not in straight line. So, f is lesser than $1.1 f_0$ but more than f_0. It is B.

I know it is ~~confusing~~ tricky. Just be aware if O and S are in straight line.

KIKY