Orbits and escape velocity

Orbits

For anything to stay in orbit it requires two things:

*A centripetal force, caused by the gravitational force acting between the object orbiting and the object being orbited

*To be moving at a high speed

We now know equations for calculating the centripetal force of an object moving in a circle of radius r AND for calculating the gravitational force between two masses separated by a distance of r.

Centripetal force at distance r: $F = mv\omega$ *or* $F = mr\omega^2$ *or* $F = \frac{mv^2}{r}$

Gravitational force at distance r: $F = \frac{GMm}{r^2}$

These forces are equal to each other, since it is the force of gravity causing the centripetal force. From these we can calculate many things about an orbiting object:

The speed needed for a given radius

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \rightarrow \frac{v^2}{r} = \frac{GM}{r^2} \rightarrow v^2 = \frac{GM}{r} \rightarrow \boxed{v = \sqrt{\frac{GM}{r}}}$$

The time of orbit for a given radius

$$mr\omega^2 = \frac{GMm}{r^2} \rightarrow \omega^2 = \frac{GM}{r^3} \rightarrow (2\pi f)^2 = \frac{GM}{r^3} \rightarrow \left(\frac{2\pi}{T}\right)^2 = \frac{GM}{r^3}$$

$$\rightarrow \frac{4\pi^2}{T^2} = \frac{GM}{r^3} \rightarrow \frac{T^2}{4\pi^2} = \frac{r^3}{GM} \rightarrow T^2 = \frac{4\pi^2 r^3}{GM} \rightarrow \boxed{T = \sqrt{\frac{4\pi^2 r^3}{GM}}}$$

Energy of Orbit

The total energy of a body in orbit is given by the equation:

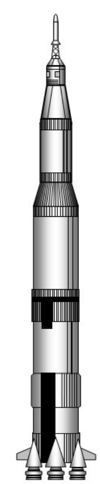
Total energy = Kinetic energy + Potential energy or $E_T = E_K + E_P$

$$E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} \rightarrow E_T = \frac{1}{2}m\left(\sqrt{\frac{GM}{r}}\right)^2 - \frac{GMm}{r} \rightarrow E_T = \frac{1}{2}\frac{GMm}{r} - \frac{GMm}{r} \rightarrow \boxed{E_T = -\frac{1}{2}\frac{GMm}{r}}$$

Geostationary Orbits

Geostationary orbiting satellites orbit around the equator from West to East. They stay above the same point on the equator meaning that the time period is 24 hours or seconds. They are used for communication satellites such as television or mobile phone signals.

Escape Velocity



For an object to be thrown from the surface of a planet and escape the gravitational field (to infinity) the initial kinetic energy it has at the surface must be equal to the potential energy (work done) to take it from the surface to infinity.

Potential energy: $E_P = m\frac{GM}{R}$ Kinetic energy: $E_K = \frac{1}{2}mv^2$

$$\frac{1}{2}mv^2 = m\frac{GM}{R} \rightarrow \frac{1}{2}v^2 = \frac{GM}{R} \rightarrow v^2 = \frac{2GM}{R} \rightarrow \boxed{v = \sqrt{\frac{2GM}{R}}}$$

For an object to be escape the Earth…..

$$v = \sqrt{\frac{2GM}{R}} \qquad v = \sqrt{\frac{2(6.67\times10^{-11})(6.00\times10^{24})}{(6.40\times10^{6})}} \qquad v = 11183 \text{ m/s}$$

This calculation is unrealistic. It assumes that all the kinetic energy must be provided instantaneously. We have multistage rockets that provide a continuous thrust.