

# Nuclear radius

Rutherford gave us an idea of the size of the nucleus compared to the atom, but more experimental work has since been done to find a more accurate measurement.

## Closest Approach of Alpha Particles

Rutherford fired alpha particles at gold atoms in a piece of foil. They approach the nucleus but slow down as the electromagnetic repulsive force become stronger.

Eventually they stop moving, all the kinetic energy has been converted into potential energy as the particles come to rest at a distance  $r$  from the centre of the nucleus.

$E_K = E_P \rightarrow E_P = qV$  where  $V$  is the electric potential at a distance of  $r$  from the centre

$$E_P = q \frac{Q}{4\pi\epsilon_0 r} \rightarrow E_K = q \frac{Q}{4\pi\epsilon_0 r} \rightarrow \boxed{r = q \frac{Q}{4\pi\epsilon_0 E_K}}$$

This gives us the upper limit of the radius of a nucleus.

Calculating the nuclear radius this way gives us a value of  $r = 4.55 \times 10^{-14}$  m or 45.5 fm (where  $1 \text{ fm} = 1 \times 10^{-15}$  m)

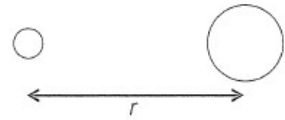
Modern measurements give us values of approximately  $r = 6.5$  fm

(Remember that 1 eV of energy is equal to  $1.6 \times 10^{-19}$  J)

particle mass  $m$   
charge  $+q$

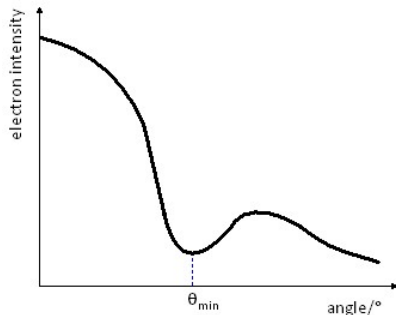


nucleus mass  $M$   
charge  $+Q$



## Electron Diffraction

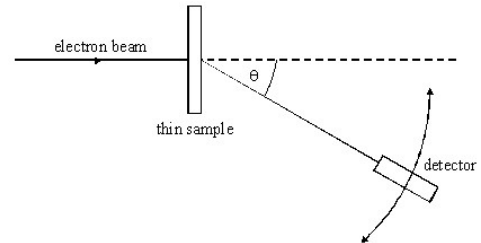
A beam of electrons was fired at a thin sample of atoms and the diffraction pattern was detected and then examined.



The graph shows a minimum at a value of  $\theta_{\min}$ . We can use this to find a value of the nuclear radius.

$$\boxed{\sin \theta_{\min} = \frac{0.61\lambda}{D}}$$

Where  $D$  is the nuclear radius and  $\lambda$  is the de Broglie wavelength of the beam of electrons. We can calculate this as follows:



The kinetic energy gained by the electrons is  $E_K = eV$  where  $e$  is the charge on the electron and  $V$  is the potential difference used to accelerate it. So we now have:

$$\frac{1}{2}mv^2 = eV \rightarrow mv^2 = 2eV \rightarrow m^2v^2 = 2meV \rightarrow \sqrt{m^2v^2} = \sqrt{2meV} \rightarrow mv = \sqrt{2meV}$$

We can now substitute this into the equation for de Broglie wavelength:  $\lambda = \frac{h}{mv} \rightarrow \boxed{\lambda = \frac{h}{\sqrt{2meV}}}$

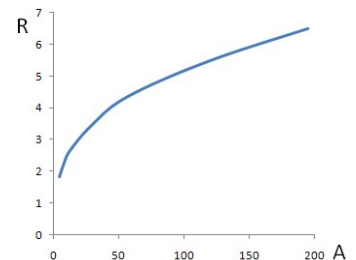
## Nuclear Radius

From the experimental results a graph was plotted of  $R$  against  $A$ . A graph like the one to the right was obtained. They saw that  $R$  depends not on  $A$ , but on  $A^{1/3}$ .

When they plotted the graph of  $R$  against  $A^{1/3}$  they found a straight line that cut the origin and had a gradient of  $r_0$ . ( $r_0$  is a constant representing the radius of a single nucleon and has a value of between 1.2 and 1.5 fm)

The radius of a nucleus has been found to be:

$$\boxed{R = r_0 A^{1/3}}$$



## Nuclear Density

Now that we have an equation for the nuclear radius we can calculate the density of a nucleus.

If we have a nucleus of  $A$  nucleons, we can assume the mass is  $Au$  and the volume is the volume of a sphere:

$$\rho = \frac{m}{V} \rightarrow \rho = \frac{Au}{\frac{4}{3}\pi R^3} \rightarrow \rho = \frac{Au}{\frac{4}{3}\pi (r_0 A^{1/3})^3} \rightarrow \rho = \frac{Au}{\frac{4}{3}\pi r_0^3 A} \rightarrow \rho = \frac{u}{\frac{4}{3}\pi r_0^3}$$

We can see that the density is independent of the nucleon number and gives a value of:  $3.4 \times 10^{17} \text{ kg m}^{-3}$ .