12. Quantum & Nuclear Physics - AHL

12.1 - The Interaction of matter with radiation

Date

Understandings

- Photons
- The photoelectric effect
- Matter Waves
- Pair production & pair annihilation
- Quantization of angular momentum in Bohr model for hydrogen
- The wave function
- Uncertainty principle for energy & time and position & momentum.
- Tunneling, potential barrier and factors affecting tunneling probability

Applications and Skills

- Discussing photoelectric effect experiment and explaining which features cannot be explained in the classical wave theory.
- Solving photoelectric problems both graphically and algebraically
- Discussing experimental evidence for matter ~~waves~~ waves, including experiment where wave nature of electrons is evident
- Stating order of magnitude estimates from the uncertainty principle.

Data Booklet Reference

- $E = hf$
- ~~$E_{max} = hf - \Phi$~~
- $E_{max} = hf - \Phi$
- $E = -\frac{13.6\text{ eV}}{n^2}$
- $mvr = \frac{nh}{2\pi}$
- $P(r) = |\Psi|^2 \Delta V$
- $\Delta x \Delta p \geq \frac{h}{4\pi}$
- $\Delta E \Delta t \geq \frac{h}{4\pi}$

Guidance

- The order of magnitude estimates from the uncertainty principle may include (but not limited to) estimates of the energy of ground state atom, the impossibility of an electron existing within a nucleus, and lifetime of electron in an excited energy state.
- Tunneling is treated qualitatively using idea of continuity of wave functions.

1) The Quantum Nature of Radiation

- Light should be thought of as a collection of quanta, or bundles of energy. Each quantum has energy E given by $E = hf$.

n = integer number of quanta h = Planck's constant f = Frequency

$E = nhf$	Planck's Hypothesis	
$h = 6.63 \times 10^{-34}$ Js		$n = 1, 2, 3 \ldots$

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$E = hF$

$E = h\left(\frac{c}{\lambda}\right)$

$E = \frac{hc}{\lambda}$	Planck's Hypothesis

If light has wavelength of 500 nm, what is the Energy contained in a single quantum?

$E = \frac{hc}{\lambda}$

$E = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{500 \times 10^{-9}}$

$E = 3.98 \times 10^{-19}$ J

- So, Planck says that thermal oscillators only absorb or emit light in chunks of E.

2) The Photoelectric Effect

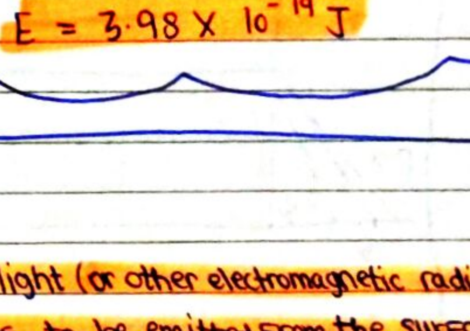
- Photoelectric effect - the phenomenon in which light (or other electromagnetic radiation) incident on a metallic surface causes electrons to be emitted from the surface.

- energy packet = photon

- Photosensitive metals emit electrons from their surface when struck by radiant energy.

- The light (incident) must do work on the electrons for this effect to happen.

- If this circuit is reversed (plates), there is 0 A on Ammeter. The electrons are not repelled

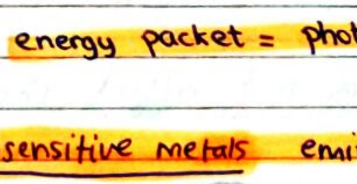


~~Electro~~ Electrons that are emitted go in circuit and can be calculated.

The - repels the electrons

- Stopping Voltage (V_s) - Voltage at which current is zero. No electrons arrive to plate.

Significance - Max. Kinetic energy (of the emitted electrons) must be eV_s.



$E_{max} = eV_s$	= W done
Max energy of emitted electrons	

Q) The stopping voltage is −0.40V. What is the max energy of emitted electrons?

(Ans) ~~0.40 × e~~ $E = eV_s = 0.40\,eV$

$0.40 \times 1.6 \times 10^{-19} = 6.4 \times 10^{-20}$ J

b) What is the max speed of emitted electrons?

(Ans) $6.4 \times 10^{-20} = \frac{1}{2}mv^2$

$\sqrt{\frac{12.8 \times 10^{-20}}{9.110 \times 10^{-31}}} = v = 3.7 \times 10^{5}\ ms^{-1}$

Principles of Photo-electric effect

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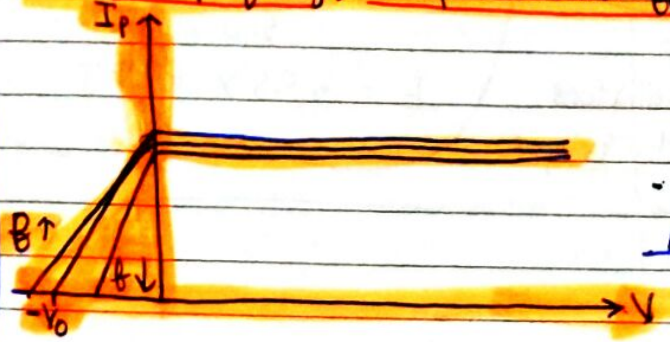
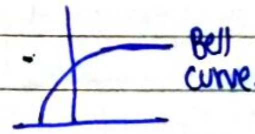
1) Stopping Voltage is independent of the intensity of light.
2) Stopping voltage depends on the light frequency.

- When the frequency of photons increases, so does the stopping voltage. V_s and Energy of released electron & frequency are proportional.
- At low frequency, no photoelectric effect.



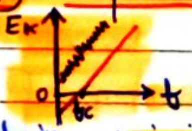


- Even if intensity is low, photoelectric effect starts.

These characteristics support particle-like behaviour.

1) The intensity of the incident light does not affect the energy of the emitted electrons.

2) The electron energy depends on the frequency of the incident light.



Straight lines.

3) There is a certain minimum frequency below which no electrons are emitted.

4) Electrons are emitted with no time-delay.

- ALL OF THE ABOVE CHARACTERISTICS ARE THE EXACT OPPOSITE OF THE CLASSICAL (OLD) THEORY.
- CLASSICAL - Light is a wave.
- Modern, quantum - Light is a particle (stream).

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$E = hf = \frac{hc}{\lambda}$	Energy of a photon

Work function Φ = minimum amount of energy needed to "knock" an electron from the metal. A frequency at least as great as cutoff frequency f_0 is needed.

$\Phi = hf_0$	Work Function

The freed electron has a maximum kinetic energy:

$E_{K,max} = hf - \phi = eV$	Maximum E_K

Putting it all together:

$hf = hf_0 + eV$ $hf = \Phi + E_{max}$	Photoelectric Effect

Example A photosensitive material has work function of 5.5eV. Find minimum frequency f_0 of light needed to free an electron from its surface.

$$5.5 = hf_0$$, convert eV to J since h has units Js.

$$\frac{5.5 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = f_0 = 1.33 \times 10^{15}\ Hz$$

$$f_0 = 1.3 \times 10^{15}\ Hz$$

Exam tip!
Any excess energy will be given to the electron as kinetic energy.

Continued... Find maximum kinetic energy of an electron freed by a photon having a frequency of 2.5×10^{15} Hz.

Solution $hf = 5eV + E_{K,max}$

$$(6.63 \times 10^{-34} \times 2.5 \times 10^{15}) - 5.5(1.6 \times 10^{-19}) = E_{K,max}$$

$$E_{K,max} = \cancel{8.6 \times 10^{-19}\ J}\ 7.8 \times 10^{-19}\ J$$

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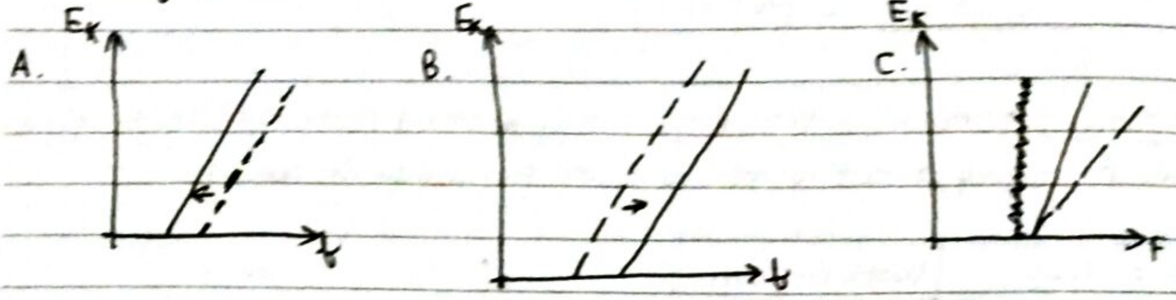
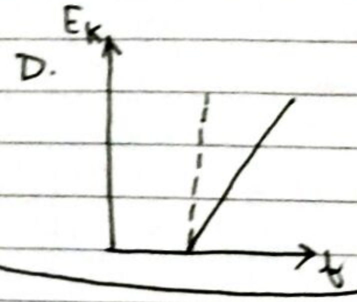
PRACTICE

Which graph shows the variation for a metal having a higher work function?





Solution. High ϕ means higher f_0. f_0 is at intercept of x-axis.

Answer : B

3) The wave nature of matter

A photon is a quantum of energy (particle) having an associated frequency (wave).

$E = hf = \frac{hc}{\lambda}$	Energy of a photon	is a statement of wave-particle-duality of light.

de Broglie hypothesized that matter should also exhibit wave-particle - duality.

4) The de Broglie Hypothesis

"Any particle having a momentum p will have a wave associated with it having a wavelength λ of $\frac{h}{p}$."

Memorize

$\lambda = \frac{h}{p} = \frac{h}{mv}$	de Broglie Hypothesis	is a statement of wave- particle duality of Matter.

RACTICE. Electron is accelerated from rest through potential difference 100V. What is its expected de Broglie wavelength?

Solution $E_k = qV = 1.6 \times 10^{-19} \times 100 = 1.6 \times 10^{-17}\,J$

$1.6 \times 10^{-17} = \frac{1}{2}mv^2$

$\sqrt{\frac{3.2 \times 10^{-17}}{9.110 \times 10^{-31}}} = v = 5.9 \times 10^{6}\ ms^{-1}$

$\lambda = \frac{6.63 \times 10^{-34}}{9.110 \times 10^{-31} \times 5.9 \times 10^{6}} = 1.2 \times 10^{-10}$ m (about diameter of an atom.)

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- Recall that diffraction only occurs if aperture is comparable to wavelength of the incident wave. Data showed that diffraction of any electron is in accordance with de Broglie's hypothesis.

Example A particle has energy E and an associated de Broglie wavelength λ. The energy E is proportional is:

A. λ^{-2} B. λ^{-1} C. λ D. λ^{2}

Sol. $\lambda = \frac{h}{mv} \rightarrow v = \frac{h}{m\lambda}$

$E_K = \frac{1}{2}mv^2$

$E_K = \frac{1}{2}m\left(\frac{h}{m\lambda}\right)^2$

$E_K = \frac{h^2}{2m\lambda^2}$

$E_K = \left(\frac{h^2}{2m}\right)\left(\frac{1}{\lambda^2}\right)$

$E_K \propto \frac{1}{\lambda^2}$

$\boxed{\text{Ans. A}}$

Example Which phenomena provides evidence for de Broglie's hypothesis? Matter is wave

A. Electron diffraction B. X-ray production
C. Line Spectra D. Nuclear Energy Levels

According to de Broglie hypothesis, matter waves are associated with:

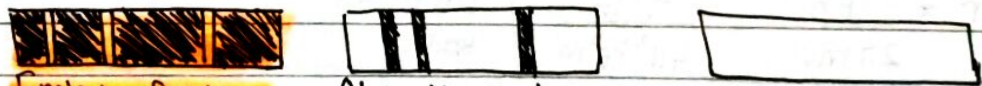
A. electrons only B. Charged particles only

C. Neutral particles only $\boxed{\text{D. All particles}}$

5) Atomic spectra and atomic energy states – review

Idea: when a low-pressure gas is subjected to a voltage, it ionizes and emits light.

Using a spectroscope, we can observe the emission spectrum. The discontinuity in the emission spectra tells us that atomic energy states are quantized.



Emission Spectrum Absorption spectrum Continuous spectrum

Q. Which provides direct evidence of the existence of discrete energy levels in an atom?

A. Continuous spectrum of light emitted by a white hot metal.

$\boxed{\text{B.}}$ Emission spectrum of gas at low pressure.

C. Emission of gamma radiation from radioactive atoms.

D. Ionization of gas atoms when bombarded by alpha particles.

6) The Bohr model of the hydrogen atom

He wanted to understand why hydrogen atom had discrete energy levels.

$E = E_K + E_P = \frac{1}{2}mv^2 + \left(-\frac{ke^2}{r}\right)$

UCM: Uniform Circular Motion of Electron

$F = \frac{ke^2}{r^2} = \frac{mv^2}{r} \rightarrow mv^2 = \frac{ke^2}{r}$

$$\boxed{E = \frac{ke^2}{2r} - \frac{ke^2}{r} = -\frac{ke^2}{2r}}$$

P.T.O.

- Now, he assumed that the angular momentum $L = mvr$, is quantized and that L can only be integral numbers n of the basic quantity $\frac{h}{2\pi}$.

$$mvr = L = \frac{nh}{2\pi}$$

$$m^2v^2r^2 = L^2 = \frac{n^2h^2}{4\pi^2}$$

$$mv^2 = \frac{n^2h^2}{4\pi^2 m r^2}$$

$mv^2 = \frac{ke^2}{r}$ From previous page,

$$\frac{ke^2}{r} = \frac{n^2h^2}{4\pi^2 m r^2}$$

$$r_n = \frac{n^2h^2}{4\pi^2 ke^2 m}$$

- This formula means only certain electronic radii are allowed in the hydrogen atom:

$r_n = \frac{n^2h^2}{4\pi^2 ke^2 m}$	$n = 1, 2, 3, \ldots$	Hydrogen Radii

$r = \frac{nh}{2\pi mv}$, Data booklet has this. But v must be simplified.

n is the orbit level.

$r = \frac{nh}{2\pi mv} = \frac{n^2h^2}{4\pi^2 ke^2 m}$	Hydrogen Radii

$mvr = \frac{nh}{2\pi}$	Angular momentum of orbiting electron in hydrogen

So, if there is question about hydrogen at end of P1 or P2, use these formulas. Otherwise no.

-

$E_n = \frac{-13.6}{n^2}$ eV	Hydrogen Energy levels

n is orbit level.

PRACTICE:

$E_1 = -13.6$ eV

$E_2 = \frac{-13.6}{4}$ eV $= -3.40$ eV

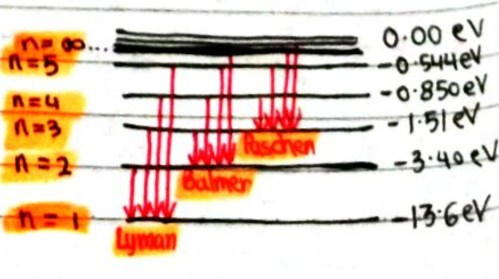
$E_3 = \frac{-13.6}{9}$ eV $= -1.51$ eV

$E_4 = \frac{-13.6}{16}$ eV $= -0.850$ eV

$E_5 = \frac{-13.6}{25}$ eV $= -0.544$ eV

The calculated values are same as table values.

Energy ↑ coz (−) sign



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7) The Schrodinger model of hydrogen atom

He argued that since electrons must exhibit wavelike properties, in order to exist in an orbit in a hydrogen atom ~~a hydrogen atom~~ a whole number of the electron's wavelength must precisely fit in the circumference of that orbit to form a standing wave.

~~Note that $n\lambda = 2\pi r_n$~~

8) The Wave Function

Schrodinger developed an equation like this:

Pretty common sense

$(E_K + E_P)\psi = E\psi$	Schrodinger's wave equation	Not in D.B.

Three things to note:

1) Built around mechanical energy conservation. $E = E_K + E_P$
2) The wavefunction ψ describes both particle & wave properties of matter simultaneously.
3) Probability cloud shows where electron has highest probability of being.

No need

$$-\left[\frac{n^2h^2}{8\pi^2 m}\right]\frac{d^2\psi}{dr^2} + 2\pi^2 m v^2 r^2 \psi = E\psi$$

Schrodinger's wave equation in 1D

Not in D.B.

Need

$P(r) = \lvert\psi\rvert^2 \Delta V$	Probability that an electron will be found within a small volume ΔV.	Important in D.B.

ΔV = ~~Potential Difference~~ volume.

The wavefunction $\psi(x,t)$ is a function of position x and time t.

$P(x,t) = \lvert\psi(x,t)\rvert^2 \Delta V$	Same formula

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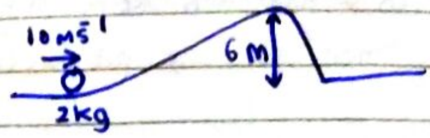
8) Tunneling

Concept

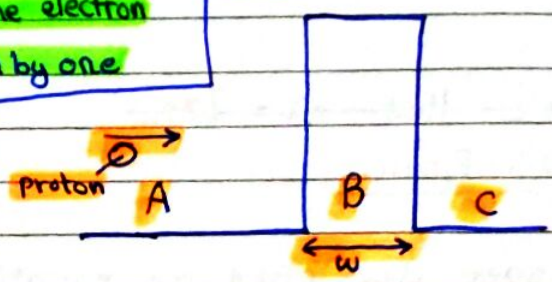
Kinetic energy of ball is 100 J.
Its potential energy at top of hill would be 120 J.
So, we say the probability of the ball to roll down on the right side is 0.
The Hill acts as a 'potential barrier'.



Context & Concept

The corresponding situation involves protons of ~~an~~ total energy E that face a region of positive electric potential. If the electron potential is V then the energy needed by one proton to go over the barrier is eV.

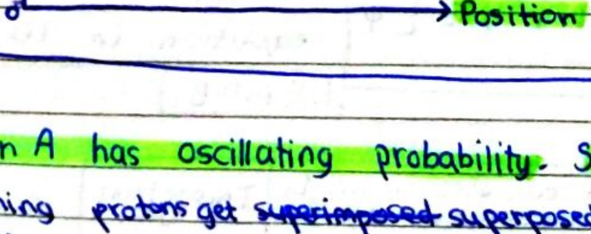
Logically, if the potential energy of the proton is below eV, ~~which~~ it cannot go into region C.
BUT, tunneling means it can.



A potential barrier to a proton.
The energy needed to go over is eV.

Tunnelling is related to particles' wave properties and is described by wavefunction

The proton has a wavefunction in each of the 3 regions. The wavefunctions must join smoothly. Region C ~~must~~ wavefunction $\neq 0$.



The wavefunction 'leaks' into the forbidden region C.
The graph shows probability for finding the electron in the regions

Region A has oscillating probability. Standing wave; the wavefunction of the incoming protons get ~~superimposed~~ superposed with reflected protons.
Region B, the probability exponentially decreases.
Region C, probability $\neq 0$. There is a wavefunction that describes the transmitted protons.
There is a small probability of finding protons in Region C.

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Three Factors affect transmission probability:

1) Mass m of the particles. Inversely — All inverse: mass, width, Energy difference
2) Width w of the barrier. Inversely
3) Difference ΔE between the energy of the barrier & that of the particles. Inversely

The larger these quantities, the small the transmission probability.

- Transmission probability for electrons > Transmission probability of protons
  "Lower mass I guess".

- Energy of particles in region A = Energy of particles in Region C
  Hence, de Broglie wavelength in region A = de Broglie wavelength in Region C

Applications of tunneling: tunnel diode, scanning tunneling microscope

9) The Heisenberg Uncertainty Principle

The basic idea behind it is wave-particle duality. ~~Sometimes~~

Tsokos Read it.

The Heisenberg uncertainty principle applied to position and momentum states that it is not possible to measure simultaneously the position and momentum of a particle with indefinite precision. This has nothing to do with imperfect measuring devices or experimental errors. It represents a fundamental property of nature. The uncertainty Δx in position and the uncertainty Δp in momentum are related by:

(Momentum form)	(Energy Form)	
$\Delta x \Delta p \geq \frac{h}{4\pi}$,	$\Delta E \Delta t \geq \frac{h}{4\pi}$	Heisenberg uncertainty principle

where h is Planck's constant. The Δ means uncertainty, not change.

Why?

Suppose you want to know the position and velocity of an electron. But this changes its momentum.
You bounce a photon off it and observe results to determine position. Then, if you bounce a second photon off it and measure time difference, you can get velocity.
Heisenburg says "It is impossible to know simultaneously an object's exact position & momentum."

If you want to know precise position, then the momentum you get has high uncertainty.

If one is more accurate, the other becomes less accurate.

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Example

An electron and a jet fighter are observed to have equal speeds of $500\ ms^{-1}$, accurate to within $\pm 0.0200\%$. What is the minimum uncertainty in the position of each if the mass of the jet is 1.00 metric ton?

$$\Delta x \Delta p \geq \frac{h}{4\pi}$$

$$\text{Uncertainty in } v = \frac{500 \times 0.02}{100} = 0.100\ ms^{-1}$$

$$\Delta x \geq \frac{h}{4\pi \Delta p}$$

$$\Delta x \geq \frac{h}{4\pi m \Delta v}$$

minimum

$$\Delta x_j \geq \frac{6.63 \times 10^{-34}}{4\pi (1000)(0.100)} \qquad \Delta x_e \geq \frac{6.63 \times 10^{-34}}{4\pi (9.110 \times 10^{-31})(0.100)}$$

$$\Delta x_j \geq 5.28 \times 10^{-37}\ m \qquad \Delta x_e \geq 5.79 \times 10^{-4}\ m$$

Much higher uncertainty for electron

$$\Delta x_j = 5.28 \times 10^{-37}\ m \qquad \Delta x_e = 5.79 \times 10^{-4}\ m$$

Example

An electron in an excited state has a lifetime of 1.00×10^{-8} seconds before it de-excites.

a) Find minimum uncertainty in energy of the photon emitted in de-excitation?

Solution

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

minimum

$$\Delta E \geq \frac{h}{4\pi \Delta t}$$

Check the "minimum" in the question. Good indication to use this formula in the end of paper 1 & 2.

$$\Delta E = \frac{6.63 \times 10^{-34}}{4\pi \times 1.00 \times 10^{-8}} = 5.28 \times 10^{-27}\ J$$

b) Find magnitude in the broadening of the frequency of the spectral line?

$$\Delta E = h \Delta f$$

$$\frac{5.28 \times 10^{-27}}{6.63 \times 10^{-34}} = \Delta f = 7.96 \times 10^{6}\ Hz$$

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10) Pair Annihilation and Pair Production

In quantum theory, you can convert matter into energy & vice versa.

- Every particle has an anti-particle with same mass (but opposite all other properties) (e.g. charge)

Pair annihilation occurs when a particle collides with its anti-particle.

~~Let's say: Collision of electron & positron. Both have E_K.~~
~~Total energy before collision is $E_T = 2(mc^2 + E_K)$~~

- During pair annihilation, matter meets antimatter both annihilate each other to become pure energy.

Pair production = a photon interacts with a nucleus to ~~produce~~ ~~release~~ a electron & positron.

Photon + nucleus → $\underbrace{\text{electron + positron}}_{\text{Pair produced.}}$

Example: A proton & antiproton are created from the void as allowed by HUP. How much time do they exist before annihilating each other?

Solution $m_p = 1.673 \times 10^{-27}$ kg

$\Delta E = \Delta m c^2$

$\Delta E = (1.673 \times 10^{-27}) \times 9 \times 10^{16} = 1.5057 \times 10^{-10}$ J

Both proton & antiproton have this energy.

Total energy $\Delta E = 3.0114 \times 10^{-10}$ J

$$\Delta E \Delta t \geq \frac{h}{4\pi}$$

$$\Delta t \geq \frac{6.63 \times 10^{-34}}{4\pi \times 3.0114 \times 10^{-10}}$$

$$\Delta t \geq 1.75 \times 10^{-25} \text{ sec}$$

$$\Delta t = 1.75 \times 10^{-25} \text{ sec}$$

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12.2 Nuclear ~~Analysi~~ Physics

Date

Understandings	Applications & Skills
• Rutherford scattering & nuclear radius • Nuclear energy levels • The neutrino • Radioactive decay & decay constant	• Describe a scattering experiment including location of minimum intensity for the diffracted particles based on their de Broglie Wavelength • Explain deviations from Rutherford scattering in high energy experiments • Describing experimental evidence for nuclear energy levels

Applications & Skills

- Solving problems with radioactive decay law for arbitrary time intervals
- Explain methods for measuring short & long half-lives

Guidance

- Students should be aware that nuclear densities are same for all nuclei and the only macro objects with this density are neutron stars

Data booklet reference

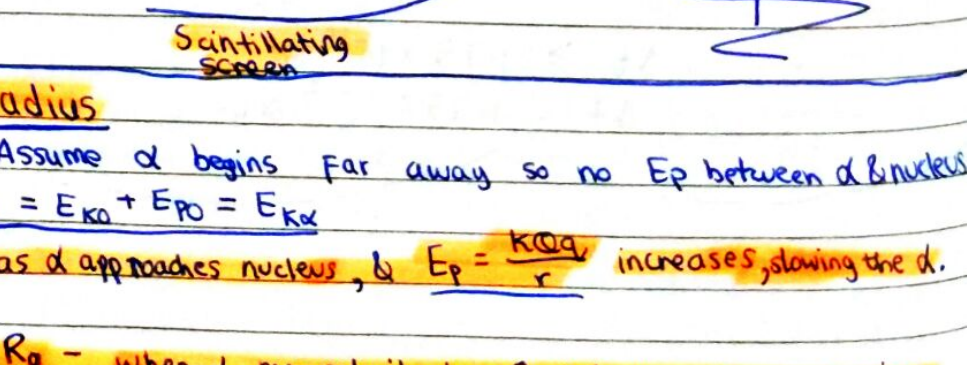
- $R = R_0 A^{1/3}$
- $\sin\theta = \frac{\lambda}{D}$
- $N = N_0 e^{-\lambda t}$
- $A = \lambda N$

1) Rutherford Scattering

Rutherford sent alpha particles through gold leaf. Very thin gold leaf!

. If we used "plum pudding" model, the alpha particles without travel without deflection straight through.

But, then Rutherford proposed that all the + charge was in center, and he called ~~it~~ the center the nucleus.



2) The Nuclear Radius

Fire α ($q = +2e$). Assume α begins far away so no E_p between α & nucleus

$$E_0 = E_{K0} + E_{P0} = E_{K\alpha}$$

But, repulsion occurs as α approaches nucleus, & $E_p = \frac{kQq}{r}$ increases, slowing the α.

Closest approach R_0 - when α momentarily has 0 velocity before reversing.

At R_0, ~~E~~ $E_K = 0$ and $\boxed{E = E_K + E_p = \frac{kQq}{r_0} = \frac{2Zke^2}{R_0}}$

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Practice.

Alpha particle has KE 2.75MeV. Approaches silicon nucleus ($Z = 14$). Find radius of silicon's nucleus.

$$2.75\,MeV = 2.75 \times 1.6 \times 10^{-19} J = 4.4 \times 10^{-13} J$$

$$R_0 = \frac{2zke^2}{E}$$

$$E = E_{Kd}$$

$$R_0 = \frac{2 \times 14 \times 8.99 \times 10^{9} \times (1.6 \times 10^{-19})^2}{4.4 \times 10^{-13}}$$

$$R_0 = \cancel{1.08 \times 10^{-57} m}\ 1.46 \times 10^{-14} m$$

Proof is beyond course, but the radius of nucleus depends on its neutrons, which has neutral charge. Thus the atomic mass no. A is used (proton number):

$R = R_0 A^{\frac{1}{3}}$	Nuclear Radius
$R_0 = 1.2 \times 10^{-15} m$	A = atomic number

Practice

Q. Find radius of gold nucleus.

$$R = 1.20 \times 10^{-15} \times 197^{\frac{1}{3}}$$
$$R = 6.98 \times 10^{-15} m$$

3) The nuclear radius - determined by diffraction

A nuclear diameter D can also be determined by measuring the diffraction of a beam of high-energy electrons or neutrons have de Broglie wavelength λ.

+ Electrons - do not respond to strong force in nucleus.
+ Neutrons - do not respond to coulomb force.

The nuclear barrier acts like a single-slit of width D. (Makes sense!) Thus,

From $\sin\theta = \frac{\lambda}{b}$

$\sin\theta = \frac{\lambda}{D}$	Nuclear Scattering

Example

Beam of 80.0 MeV neutrons are diffracted upon passing through a thin lead foil. First minimum in diffraction pattern is at 12.6°. Estimate diameter of nucleus.

$$\sin\theta = \frac{\lambda}{D}$$

We need to find λ.

$$E_k = \frac{p^2}{2m}$$

$$\cancel{E_k = \frac{mv^2}{2}}$$

$$(80 \times 1.6 \times 10^{-13}) \times 2(1.675 \times 10^{-27}) = p^2$$

$$p = 2.071 \times 10^{-19}$$

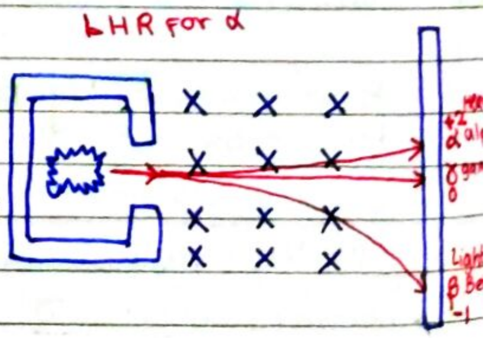
$$\lambda = \frac{h}{p}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{2.071 \times 10^{-19}} = 3.20 \times 10^{-15}\ m$$

$$D = \frac{\lambda}{\sin\theta} = \frac{3.20 \times 10^{-15}}{\sin 12.6} = 1.47 \times 10^{-14}\ m$$

4) Radioactivity

LHR for α



If a radioactive substance is place in a lead chamber and its emitted particles passed through a magnetic field, there are 3 types of radioactivity substances that can be distinguished.

$^4_2\alpha$ $\quad$ $^{0}_{-1}\beta$ $^{0}_{+1}\beta$ $\quad$ $^0_0\gamma$

Beta are electrons from the nucleus.

Gamma rays are photons with no charge.

Alpha α Decay

All α have an energy of 5 MeV.

$$^{241}Am \rightarrow {}^{237}Np + {}^4\alpha$$

High ionization harms living tissue & cells.

Date

Beta β Decay

β⁻ Decay

Neutron becomes proton and electron is emitted from the nucleus.

$${}^{14}_{6}C \rightarrow {}^{14}_{7}N + \bar{\nu}_e + {}^{0}_{-1}e^-$$

Carbon → Nitrogen + Antineutrino + electron

β⁺ Decay

Proton becomes neutron. Antielectron or Positron is emitted from nucleus.

$${}^{10}_{6}C \rightarrow {}^{10}_{5}B + \nu_e + {}^{0}_{1}e^+$$

Carbon → Boron + neutrino + positron

- Since β particles can have a large variety of kinetic energies, a neutrino was created to carry the additional E_K to conserve energy (balance energy on sides).

Gamma decay γ

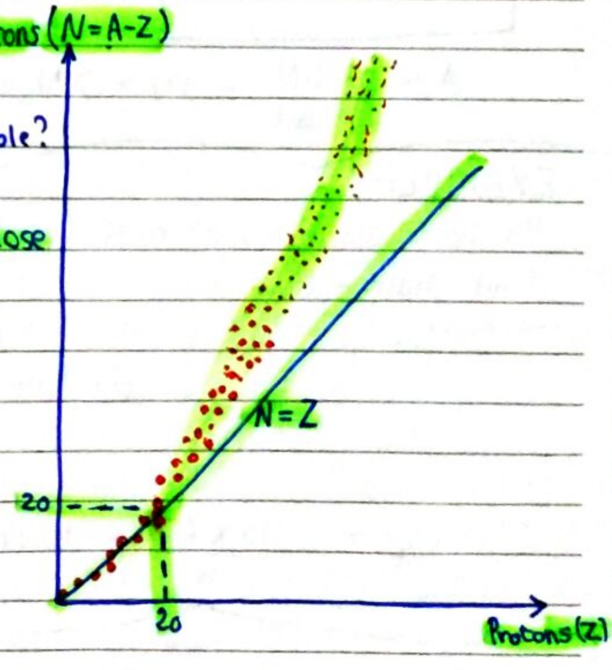
Electrons moving from an excited to de-excited state release a photon. Same happens with nuclei. They release a photon when they de-excite. The process is called gamma (γ) decay.

$${}^{234}Pu^* \rightarrow {}^{234}Pu + \gamma$$

5) Nuclear Stability

Why are some nuclei stable and others unstable?

- Upto Z=20, the proton:neutron ratio is close to 1:1.
- Beyond this, the neutrons are more than protons and this increases with atomic number.
- Extra neutrons = extra strong force
  Strong force > Repulsive Coulomb Force
- This makes the nuclides unstable.



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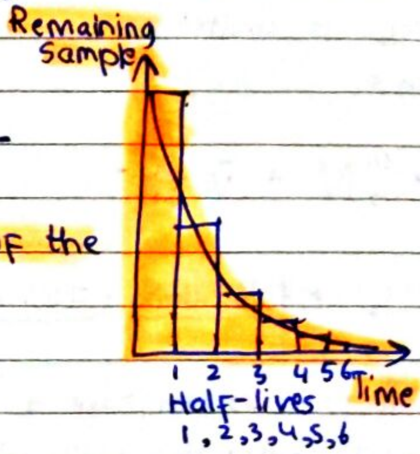
Radioactive half-life

- The decay process is random.
- But, a certain proportion will decay in a certain time. High probability of this.



Decay rate decreases exponentially.

- Radioactive half-life - time taken for half of the population of an unstable nuclide to decay.

$N = N_0 e^{-\lambda t}$	Law of radioactive decay

λ = decay constant N_0 = initial population

Decay constant = probability of decay per unit time.

Remember

$T_{\frac{1}{2}} = \frac{\ln 2}{\lambda}$	Decay constant & Half-life Memorize	Not in D.B.

- Activity = number of decays per ~~second~~ unit time. Rate of decay. Measured in (Becquerel) s^{-1}

$A = \lambda N_0 e^{-\lambda t}$	Decay rate or activity

$A = A_0 e^{-\lambda t}$

$$A = -\frac{\Delta N}{\Delta t} = \lambda N = \lambda N_0 e^{-\lambda t}$$

EXAMPLE

You have 64 grams of radioactive material that decays into 1 gram in 10 hours. Find half-time.

$\frac{64}{2^6}$, six half lives (Just write or do in head)

$$t_{half} = \frac{10 \times 60}{6} = 100 \text{ minutes}$$

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Example A nuclide X has half-life 10s. On decay, a stable nuclide Y is formed. Initially, a sample contains only the nuclide X. After what time will 87.5% of sample decay into Y?

$$N = N_0 e^{-\lambda t}$$

$$0.125 N_0 = N_0 e^{-\lambda t}$$

$$0.125 = e^{-\lambda t}$$

$$\lambda = \frac{\ln 2}{T_{\frac{1}{2}}} = \frac{\ln 2}{10} = \cancel{0.693 \text{ becquerel}}\ 0.693$$

$$0.125 = e^{-0.693t}$$

$$\frac{\ln 0.125}{-0.693} = t$$

$$t = \cancel{3.00}\ 30.0 \text{ seconds}$$

OR, $100\% \rightarrow \underbrace{50\%}_{1} \rightarrow \underbrace{25\%}_{2} \rightarrow \underbrace{12.5\%}_{3}$

$10 \times 3 = 30$ seconds

Example Element P has half-life 30 days & Q has half-life 20 days. Initially, there are equal numbers of each element.
What is the ratio $\frac{\text{no. of P atoms}}{\text{no. of Q atoms}}$ after 60 days?

$$P_0 = Q_0 = X$$

After 60 days. $P = \frac{1}{4}X$

$Q = \frac{1}{8}X$

$$\frac{P}{Q} = \frac{\frac{1}{4}}{\frac{1}{8}} = \boxed{2}$$

The initial activity of 25 µg sample is A_0. Half-life of isotope is $T_{1/2}$. What gives the initial activity & half-life of 50 µg of this isotope?

	Activity	Half-Life
A.	A_0	$T_{\frac{1}{2}}$
B.	$2A_0$	$T_{\frac{1}{2}}$
C.	A_0	$2T_{\frac{1}{2}}$
D.	$2A_0$	$2T_{\frac{1}{2}}$

$\boxed{\text{Ans. B}}$

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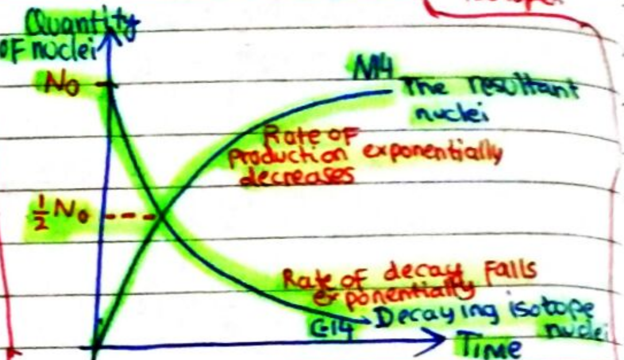
What changes after the half-life of an ~~ato~~ isotope in a sample box?

A. Mass of sample — Constant
B. Number of atoms in sample — Constant
C. Nuclei in sample — Constant

The nuclei are just converted to nuclei of another element or isotope.

D. Activity of the isotope in sample



Q) Living trees have carbon-14 atoms. Activity per gram of carbon from a living tree is 9.6 disintegrations per minute. The activity per gram of carbon in burnt wood found at an ancient campsite is 2.1 disintegrations per minute.

i) The half-life of carbon is 5500 years. Calculate decay constant & then estimate age of carbon found in campsite.

$$\text{Decay constant} = \lambda = \frac{\ln 2}{5500} = 0.000126\ \text{years}^{-1}$$ ~~= 6624 minute~~

$$A = \lambda N_0 e^{-\lambda t}$$

~~9.6 = 0.000126~~

For living tree, $A = 9.6$ per min   For the campsite burnt wood, 2.1 per min

$$9.6 = \lambda N_0 = A \qquad 2.1 = \lambda N_0 e^{-\lambda t}$$

$$N_0 = \frac{9.6 \times 24 \times 60 \times 365}{0.000126} = 4.00457 \times 10^{10} \text{ atoms}$$

A_0 = $A = A_0 e^{-\lambda t}$

$2.1 = 9.6\, e^{-\lambda t}$

Convert to year. $1103760 = 5045760 e^{-\lambda t}$

$$60 \times 24 \times 365 \times 2.1 = \cancel{0.000126 \times 4.00457 \times 10^{10}} \times e^{-0.000126 t}$$

$$\ln \frac{1103760}{5045760} = -\lambda t$$

$$\cancel{t = \frac{\ln\left(\frac{4.3619 \times 10^{-7}}{32}\ 7/32\right)}{-0.000126}}$$

$t = 12062$ years

$= 12000$ years

- So we compared the A of fresh tree & dead tree to find dead tree's age
- We converted the activities to per year from per minute.

But, the activity for over 20000 years would be so small that the probability of decay is unreliable.

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- **Isotopes** = isotopes of an element have the same number of protons & electrons, but different number of neutrons.

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